Bridging the Gap between Theory and Simulation in the Bottleneck Model

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Outline

Transport Simulators: Gap to the Theory

The Bottleneck Model in Simulators: Inspiration

- **Bottleneck model** (Vickrey, 1969; Arnott, de Palma, Lindsey, 1990s): single-road, alpha-beta-gamma generalized cost
- **Transport simulators:** tools to evaluate transport policies in large-scale scenarios
- MATSim and METROPOLIS use **bottleneck congestion**: flows are limited by road capacity
- SimMobility and METROPOLIS use **alpha-beta-gamma generalized cost** with departure-time choice
- Apart from these inspirations, transport simulators are **complex black boxes**
- To what extent are their results in line with theory?

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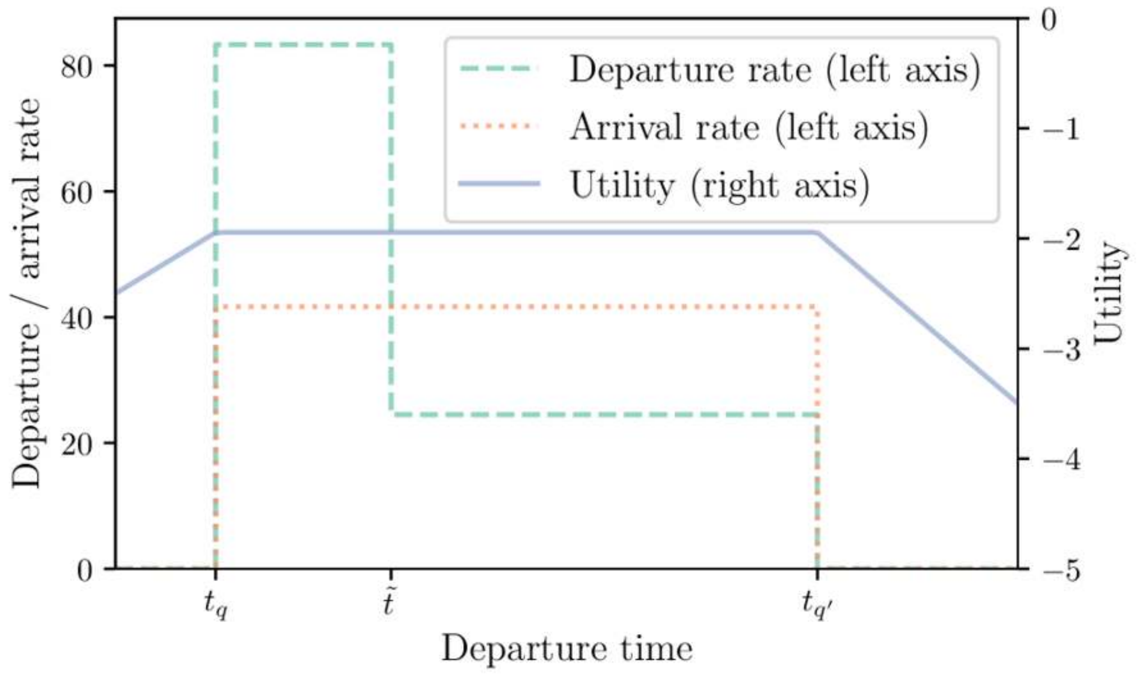
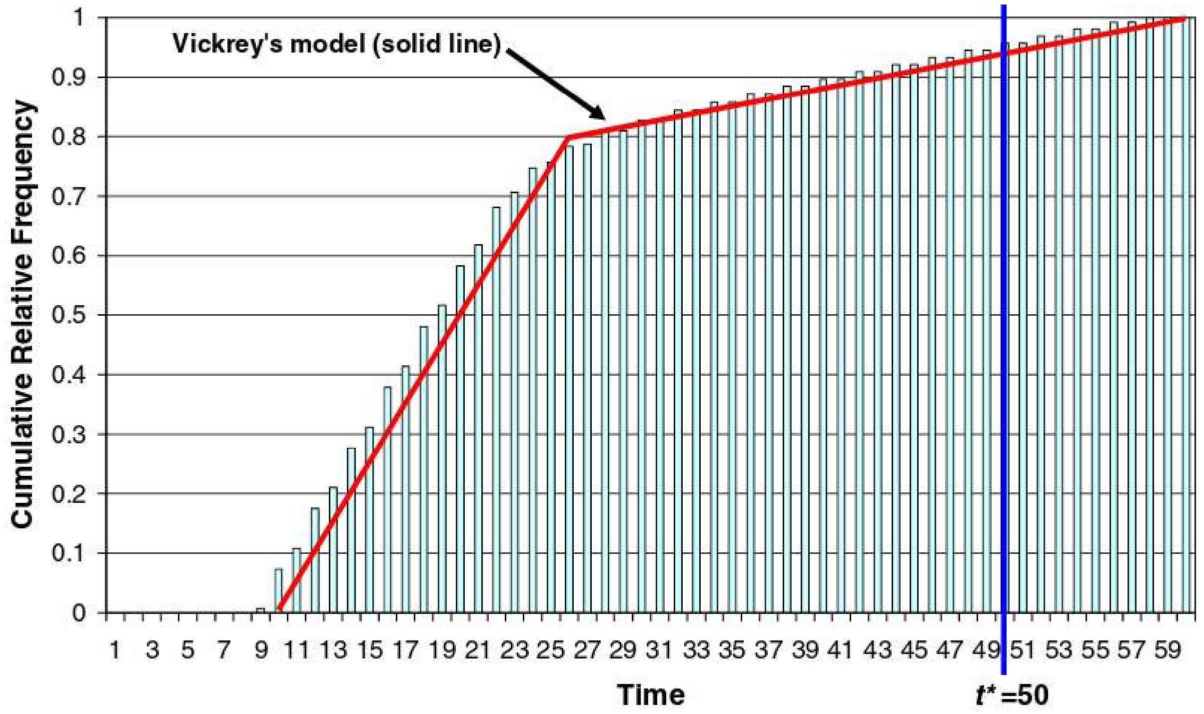
Analytical Model vs Simulation

	Analytical model	Simulation
Population	Continuum of individuals	Discrete agents
Choice model	Deterministic	Random-utility model
Behavioral representation	Continuous, implicitly-defined	Piecewise-linear, numerical approximation





Attempts at Simulating the Bottleneck Model

- **Otsubo and Rapoport (2008)**
  - Discrete agents and mixed-strategy equilibrium
  - "We report significant discrepancies in travel costs and distributions of departure time between the two solutions that slowly decrease as the number of commuters increases."
  - Methodology limited to very simple networks
- **Guo, Yang, Huang (2018)**
  - Continuum of individuals, deterministic choice model
  - "We theoretically prove that, in the simplest standard bottleneck model [...], a dynamic user equilibrium (DUE) cannot be reached through a day-to-day evolution process of travelers’ departure rate"

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Contributions

- We propose a **methodology able to replicate the analytical results of the bottleneck model**
- **Framework:** discrete agents, random-utility model
- **Solution:**
  - Continuous departure-time choice (de Palma, Ben-Akiva, Lefèvre and Litinas, 1983)
  - Continuous-time model
  - Optimal at each iteration
- This methodology is then extended into a large-scale transport simulator: **METROPOLIS2**

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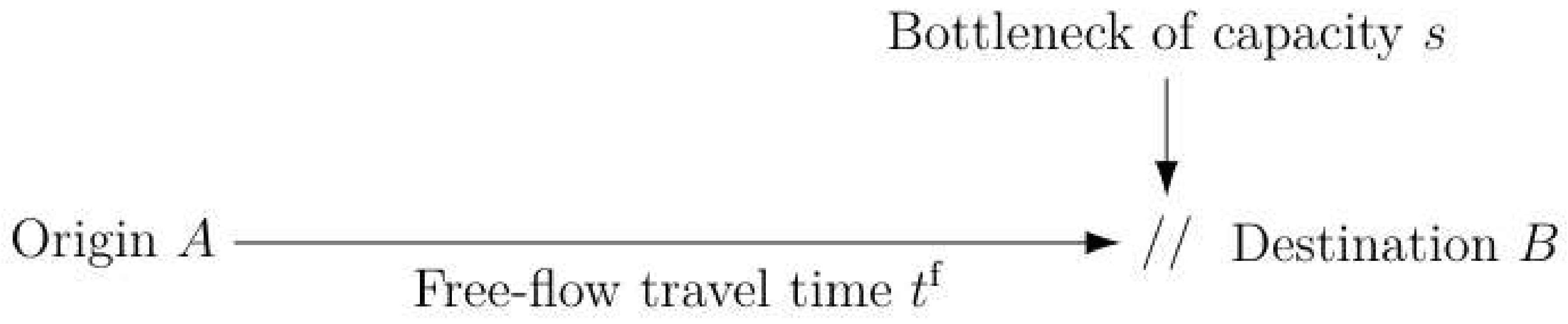
Methodology

The Bottleneck Model

- Single-road network:



- Travel time from A to B is

$$t^f + \frac{Q(t + t^f)}{s}$$

  with Q the length of the bottleneck queue at time t.

- N *discrete* agents are traveling from Origin A to Destination B
- Utility (≈ generalized cost) is given by

$$V(t) = -\alpha \cdot T(t) - \beta \cdot [t^* - t - T(t)]_+ - \gamma \cdot [t + T(t) - t^*]_+$$

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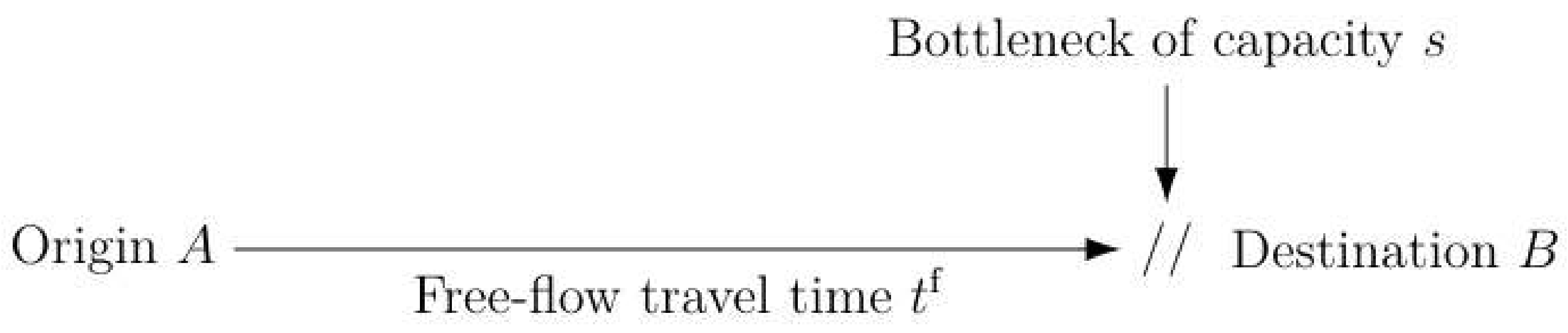

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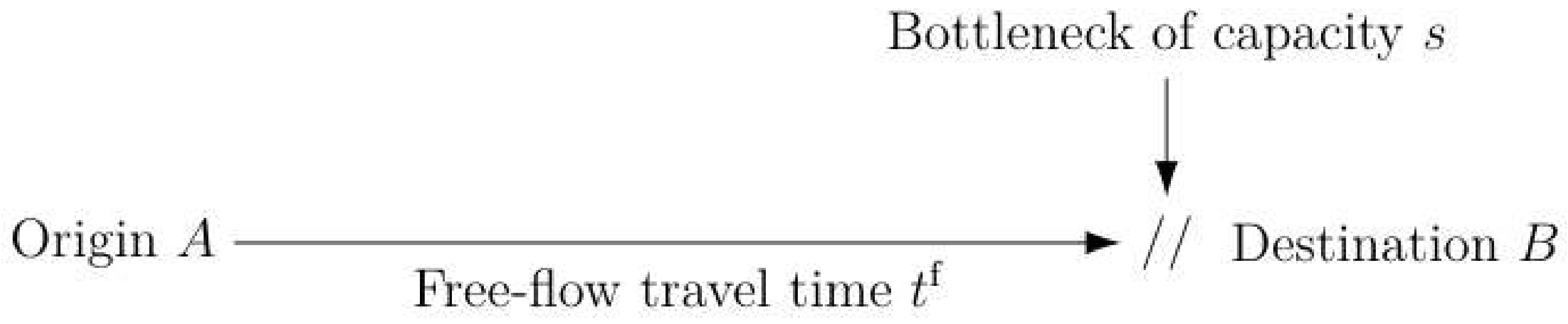

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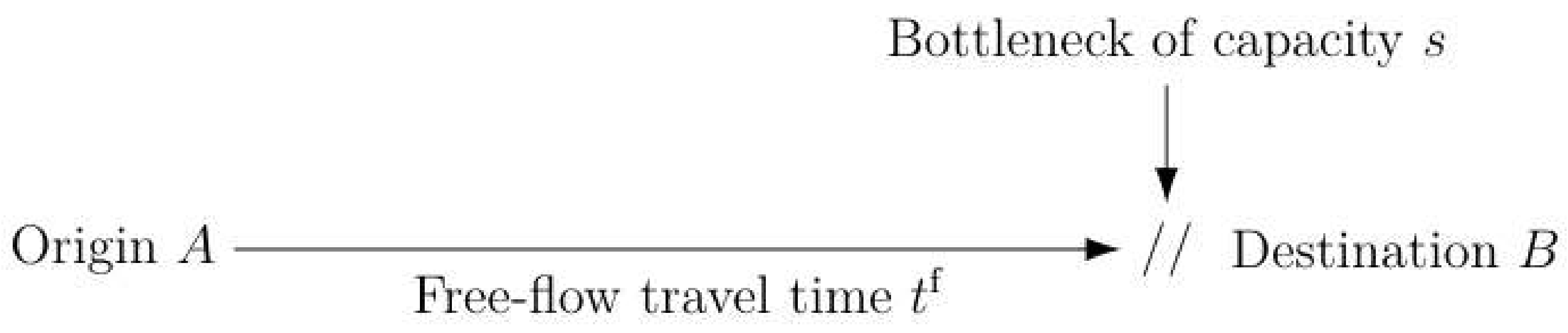

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The Bottleneck Model: Departure-Time Choice

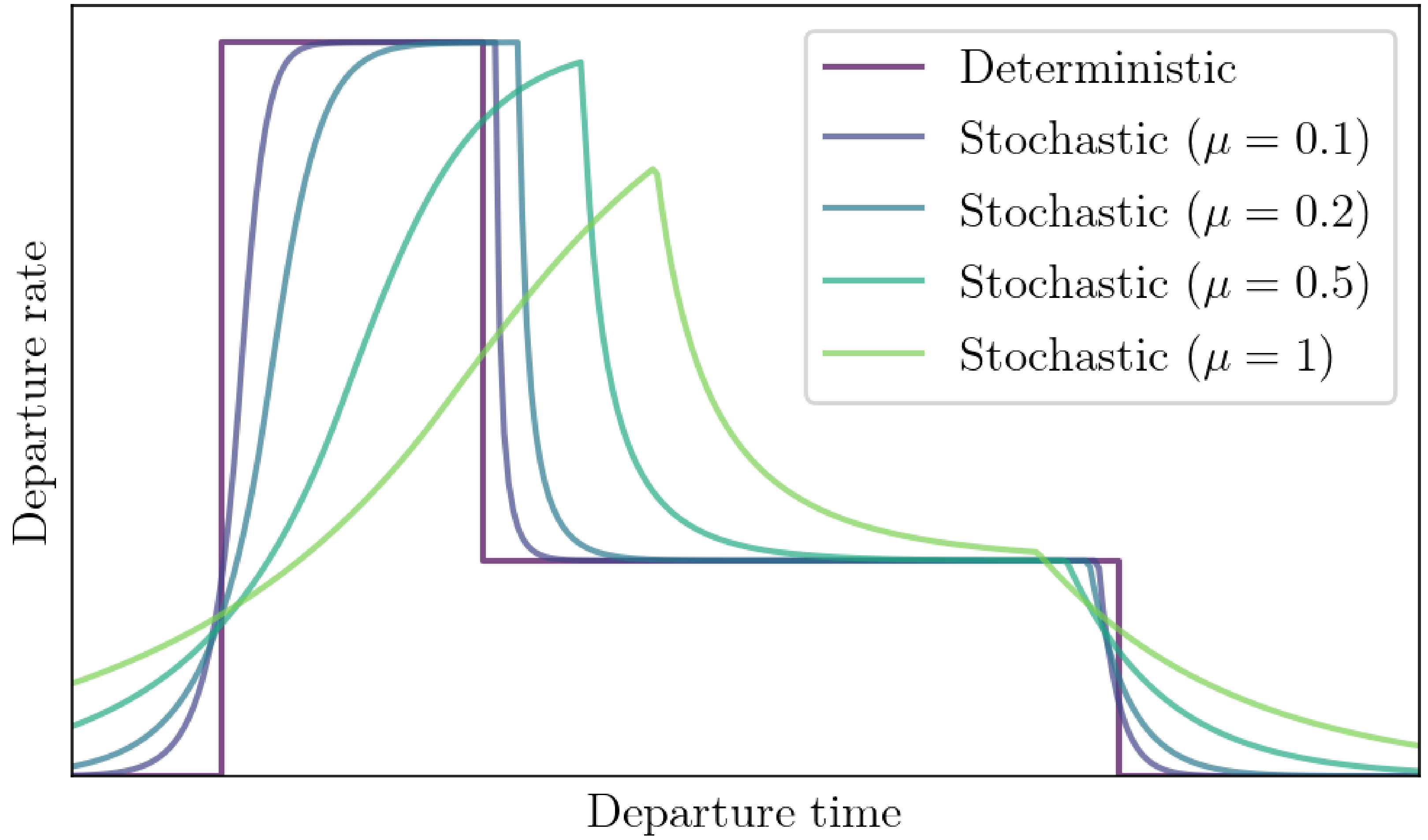
- Agents choose the departure time maximizing deterministic utility (Arnott, de Palma and Lindsey, 1990):

$$t^d = \arg\max V(t)$$

- Agents choose departure time t with probability (Continuous Logit, de Palma, Ben-Akiva, Lefèvre and Litinas, 1983):

$$p(t) = \frac{e^{V(t)/\mu}}{\int_{t^0}^{t^1} e^{V(\tau)/\mu} d\tau}$$

- **Fixed-point problem:** $p(t) \leftrightarrow V(t)$
- Alternative interpretation: **mixed strategy**



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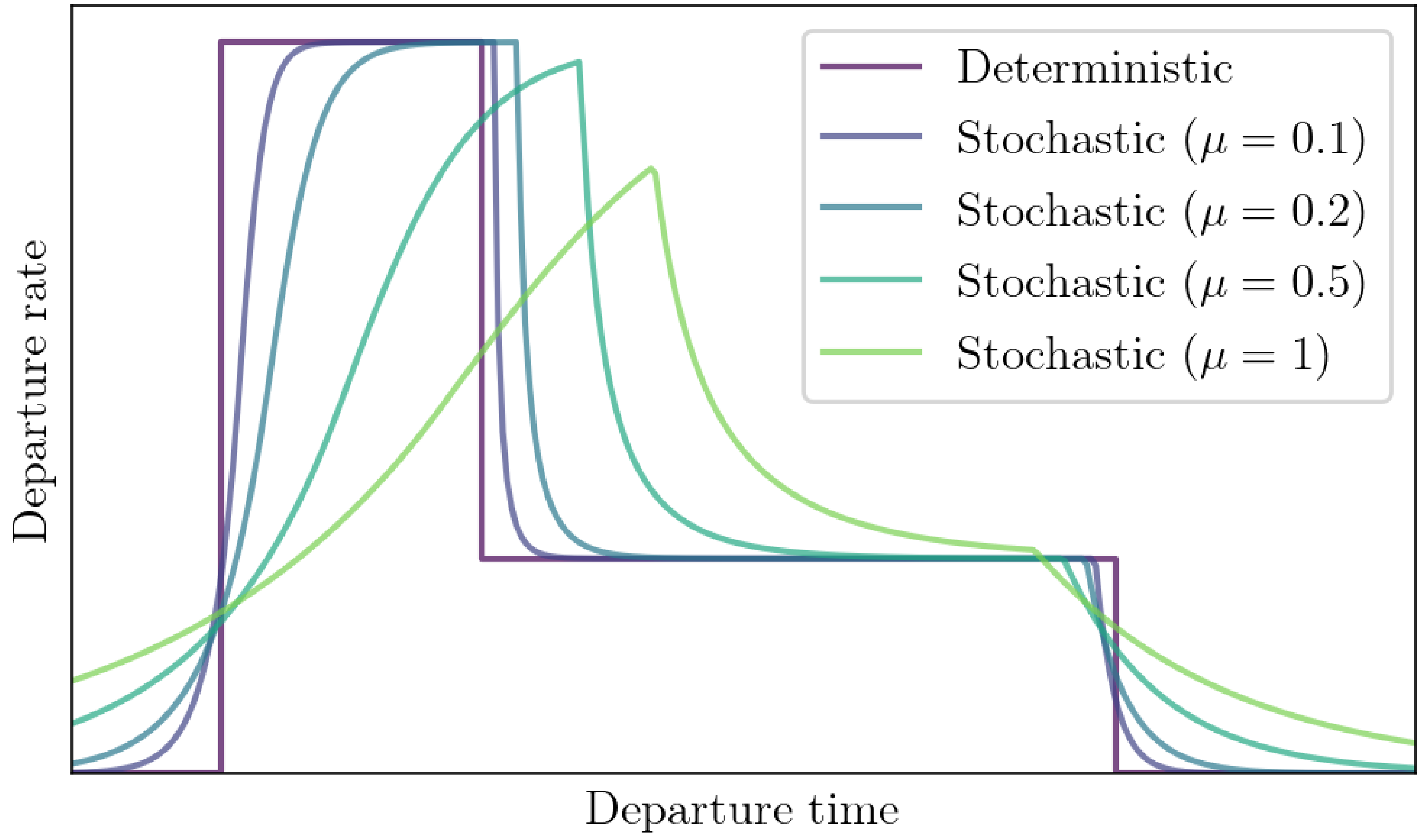

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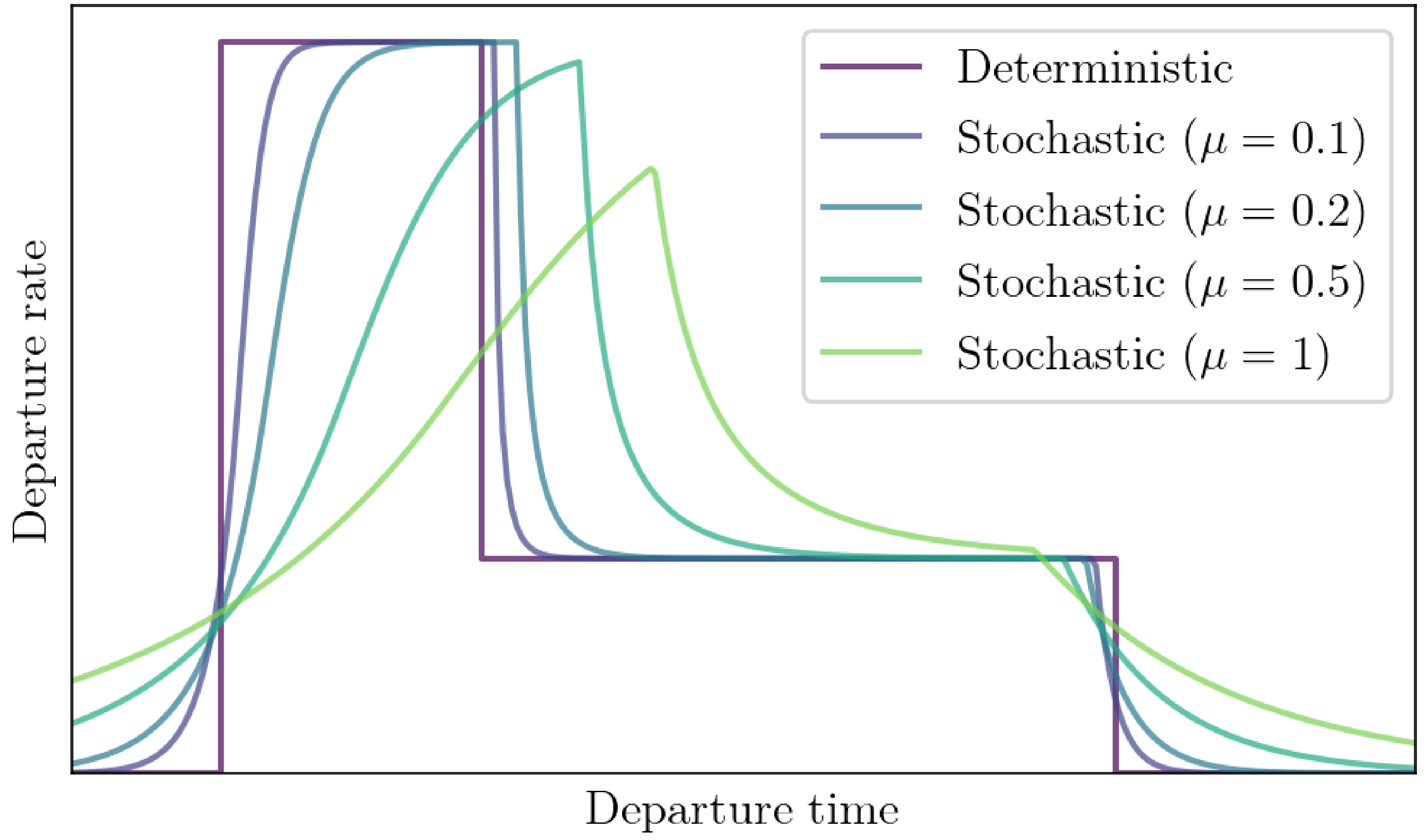

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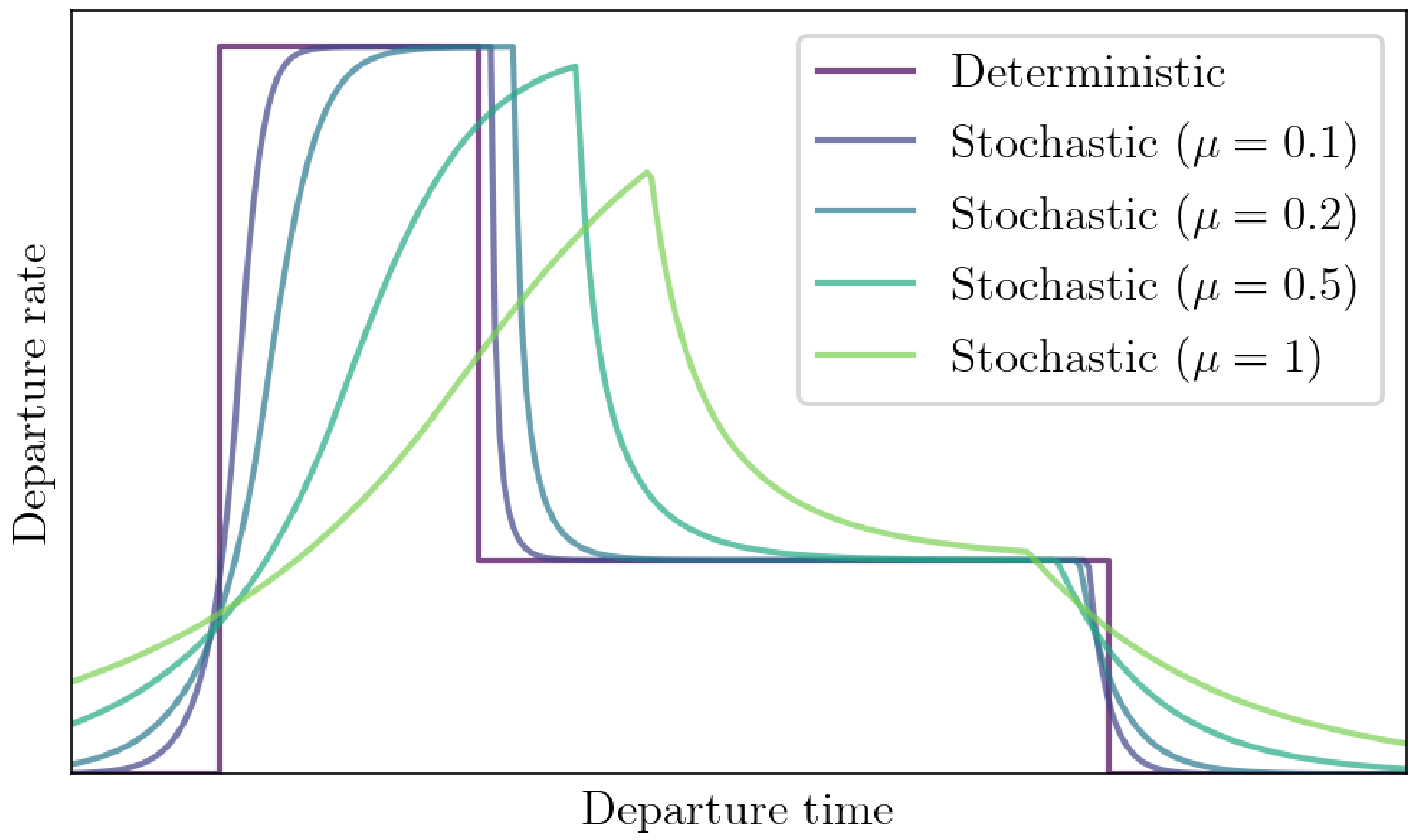
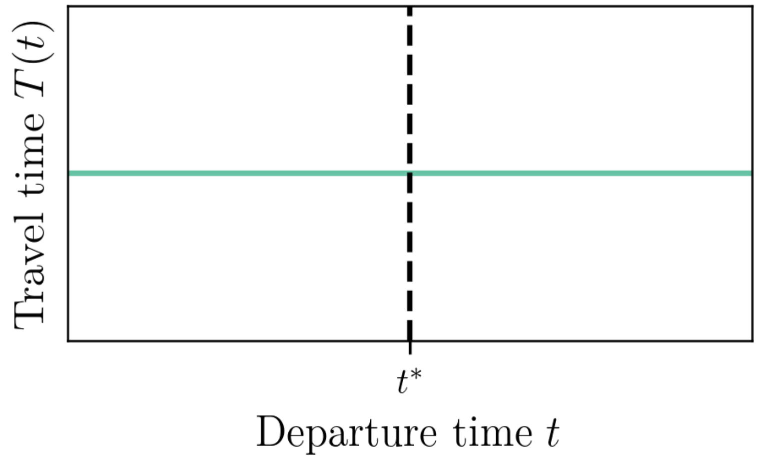


Iterative Process

Initial conditions



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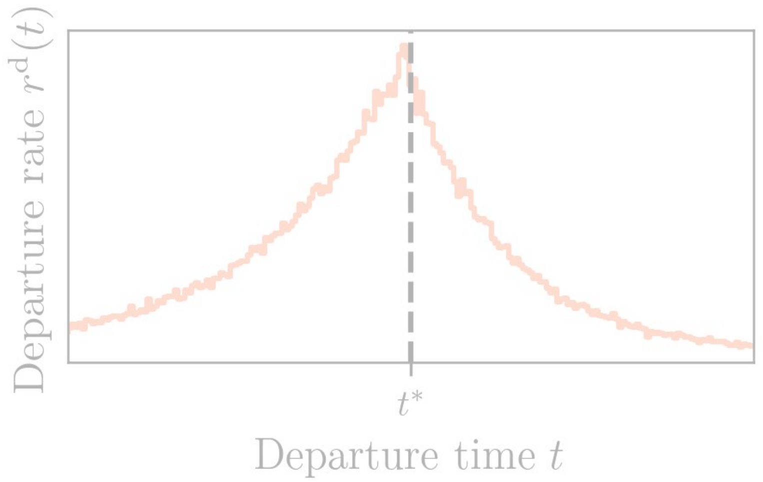
Demand model ⟶



⟶ Supply model

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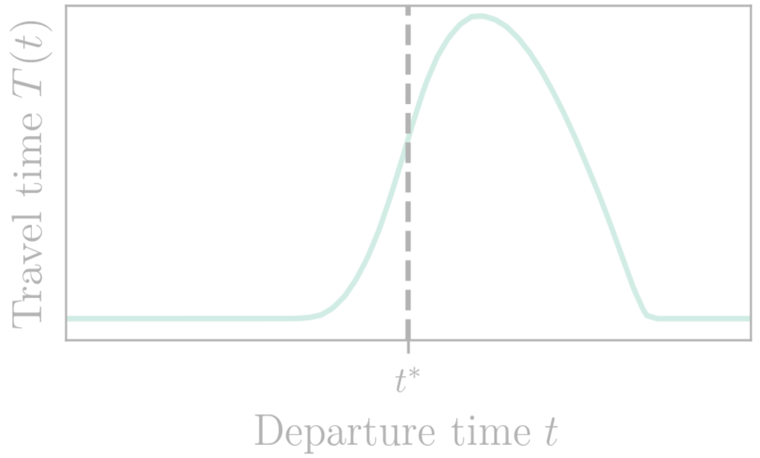

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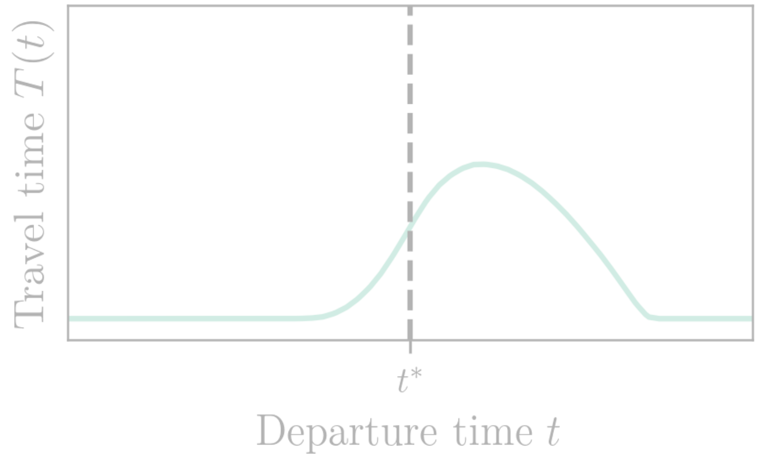

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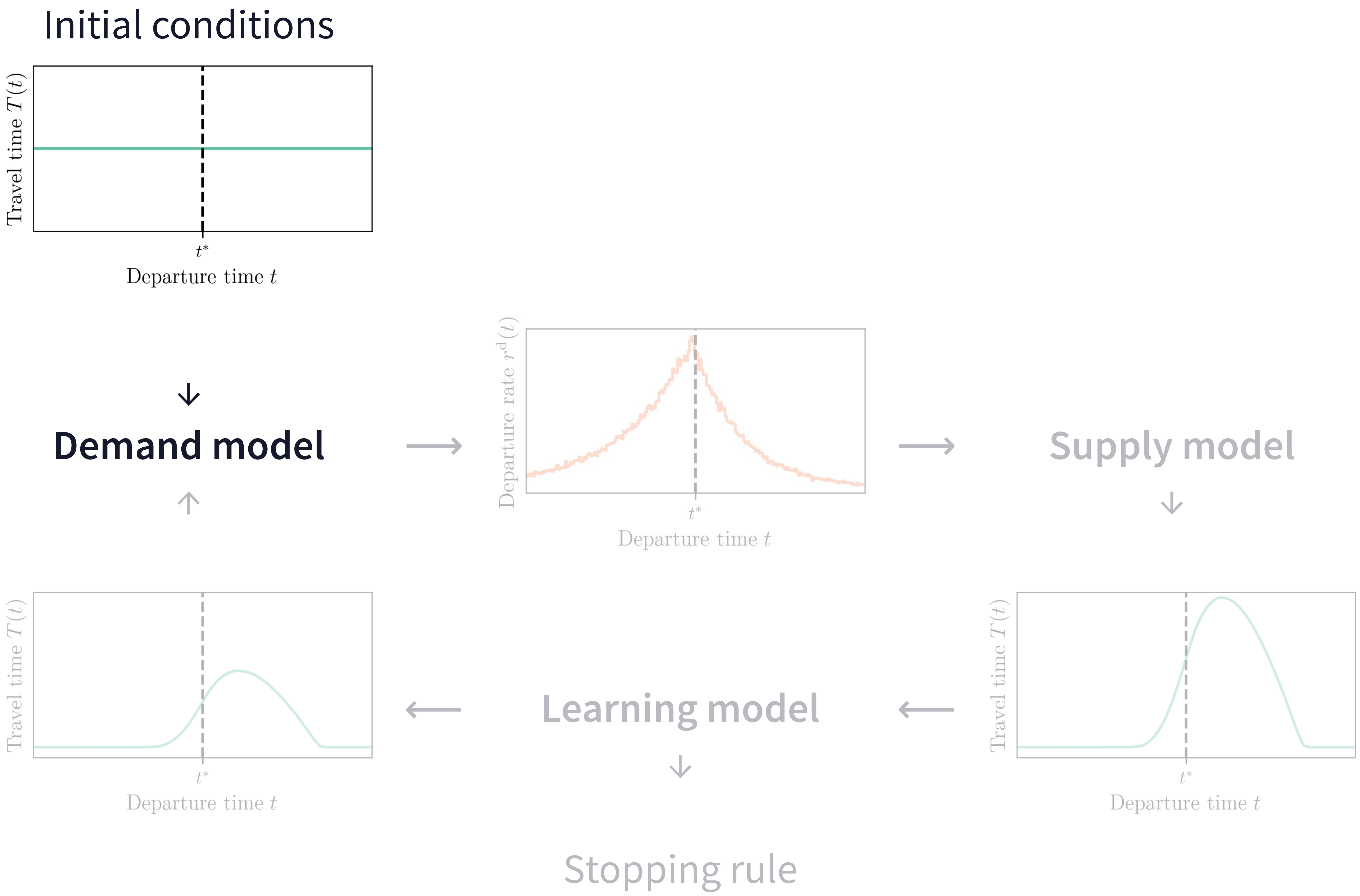
Stopping rule

Iterative Process



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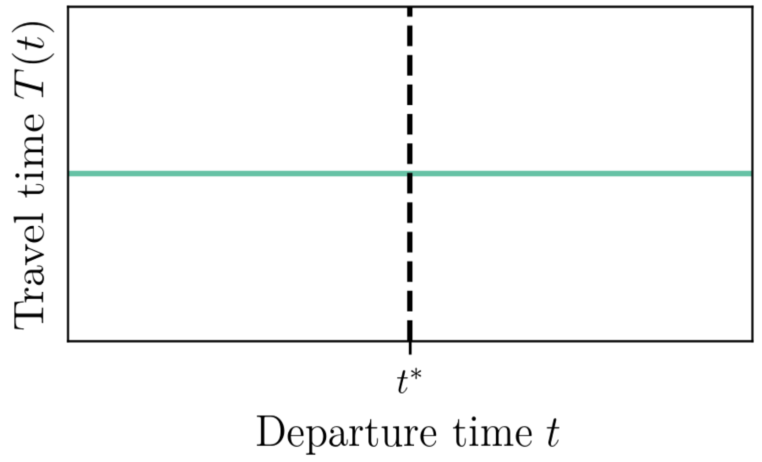

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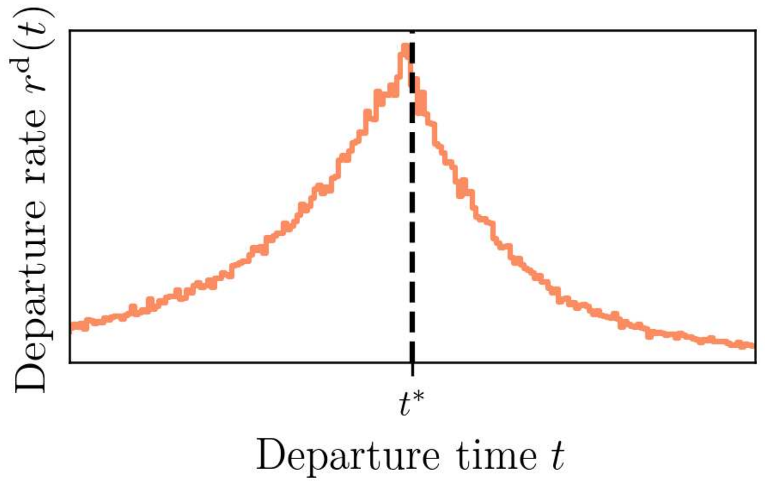

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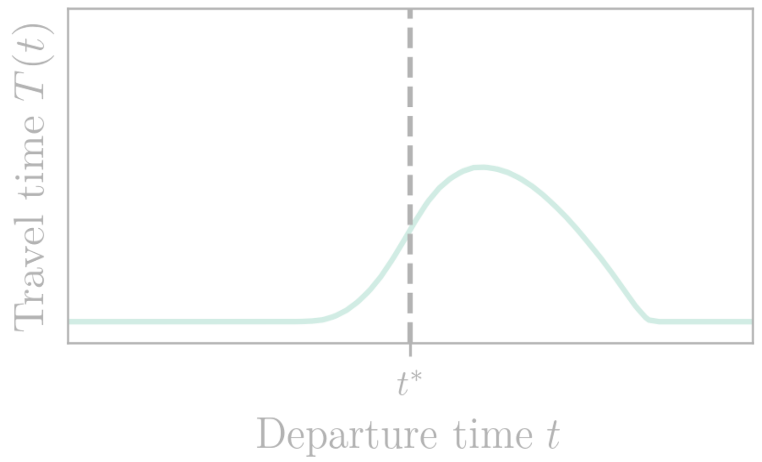

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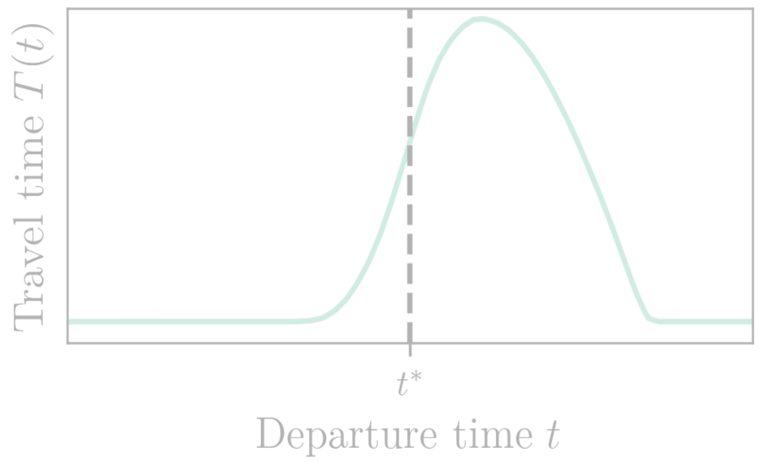

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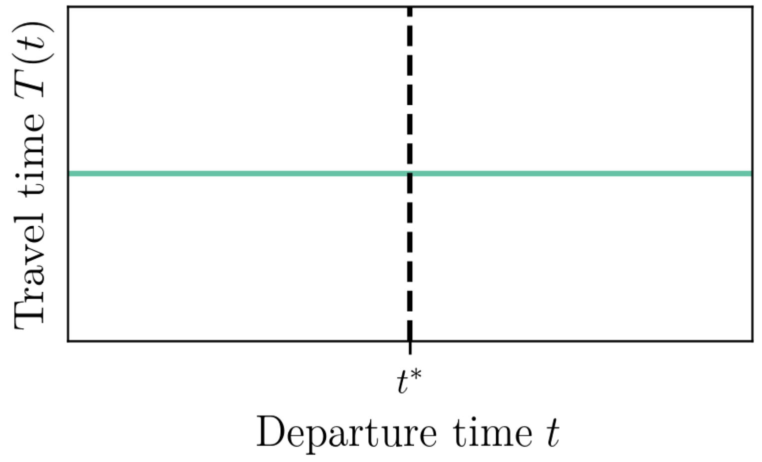

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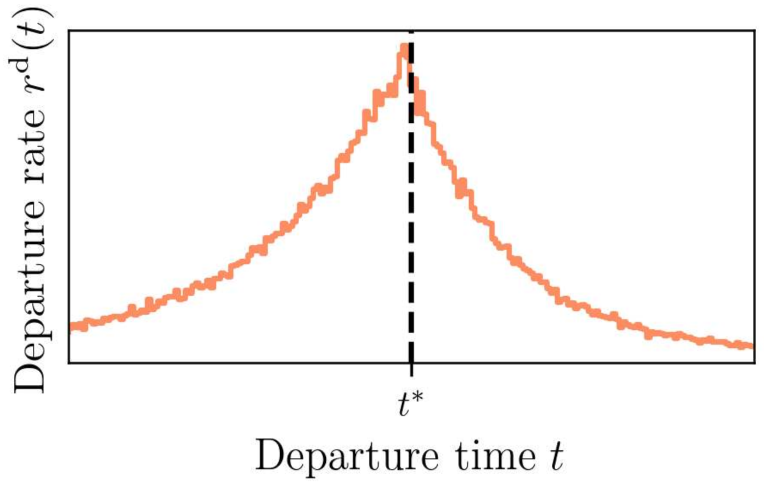

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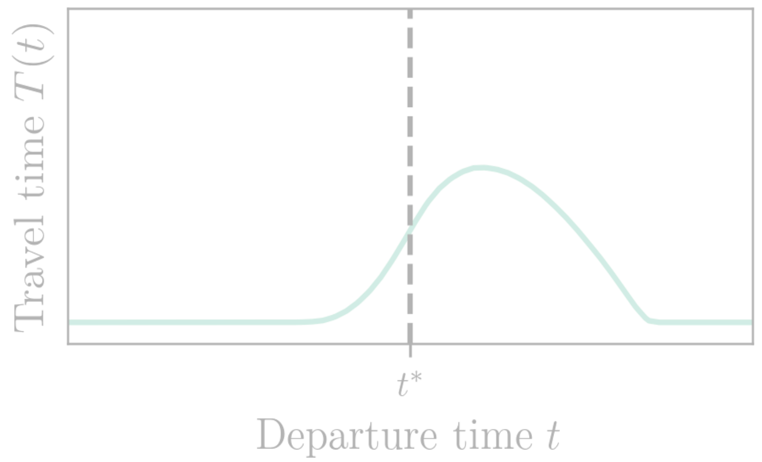

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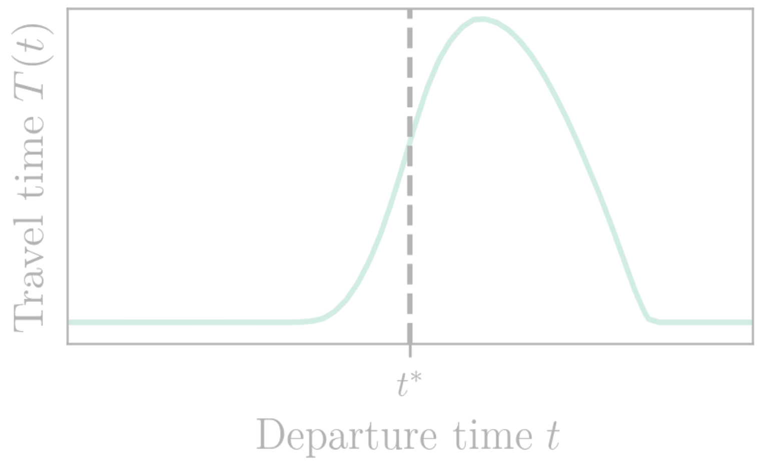

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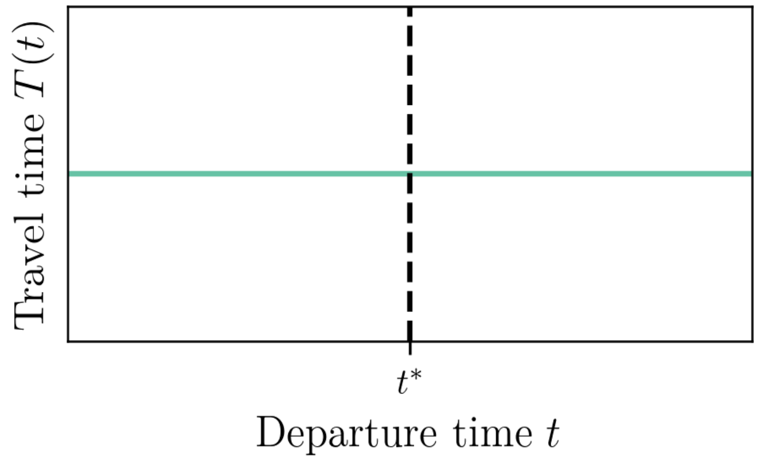

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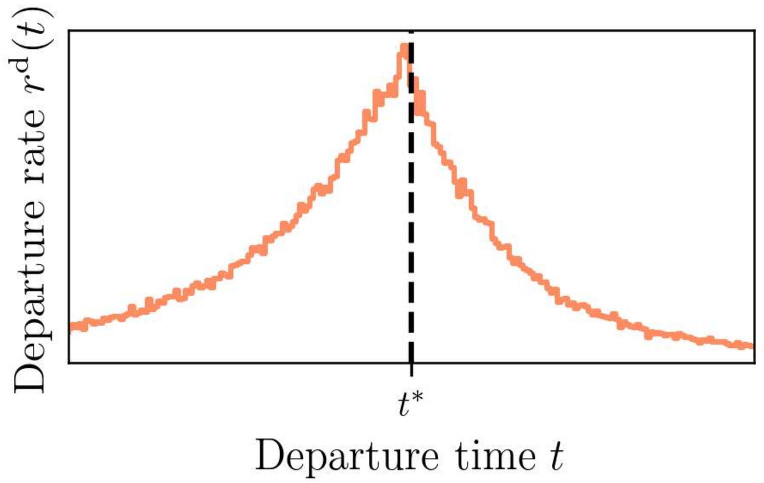

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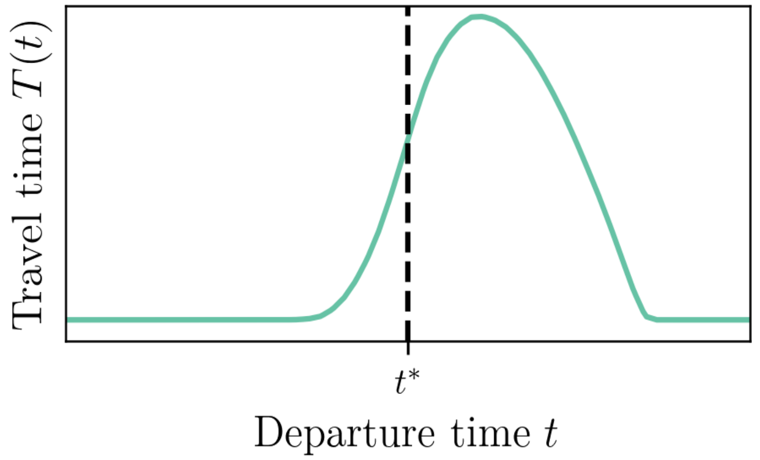

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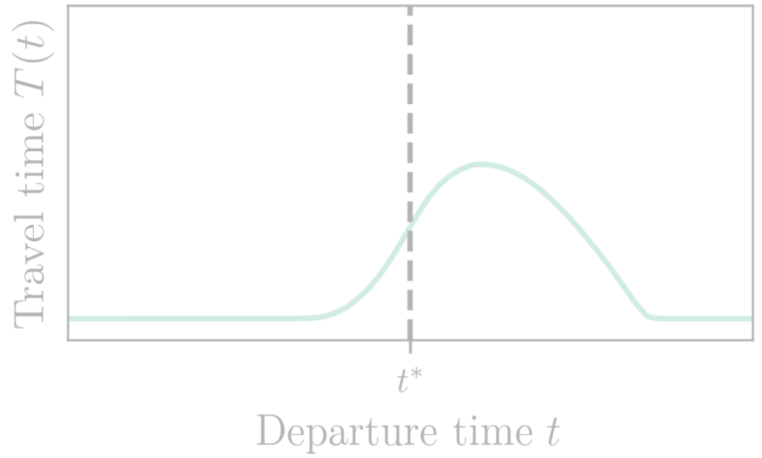

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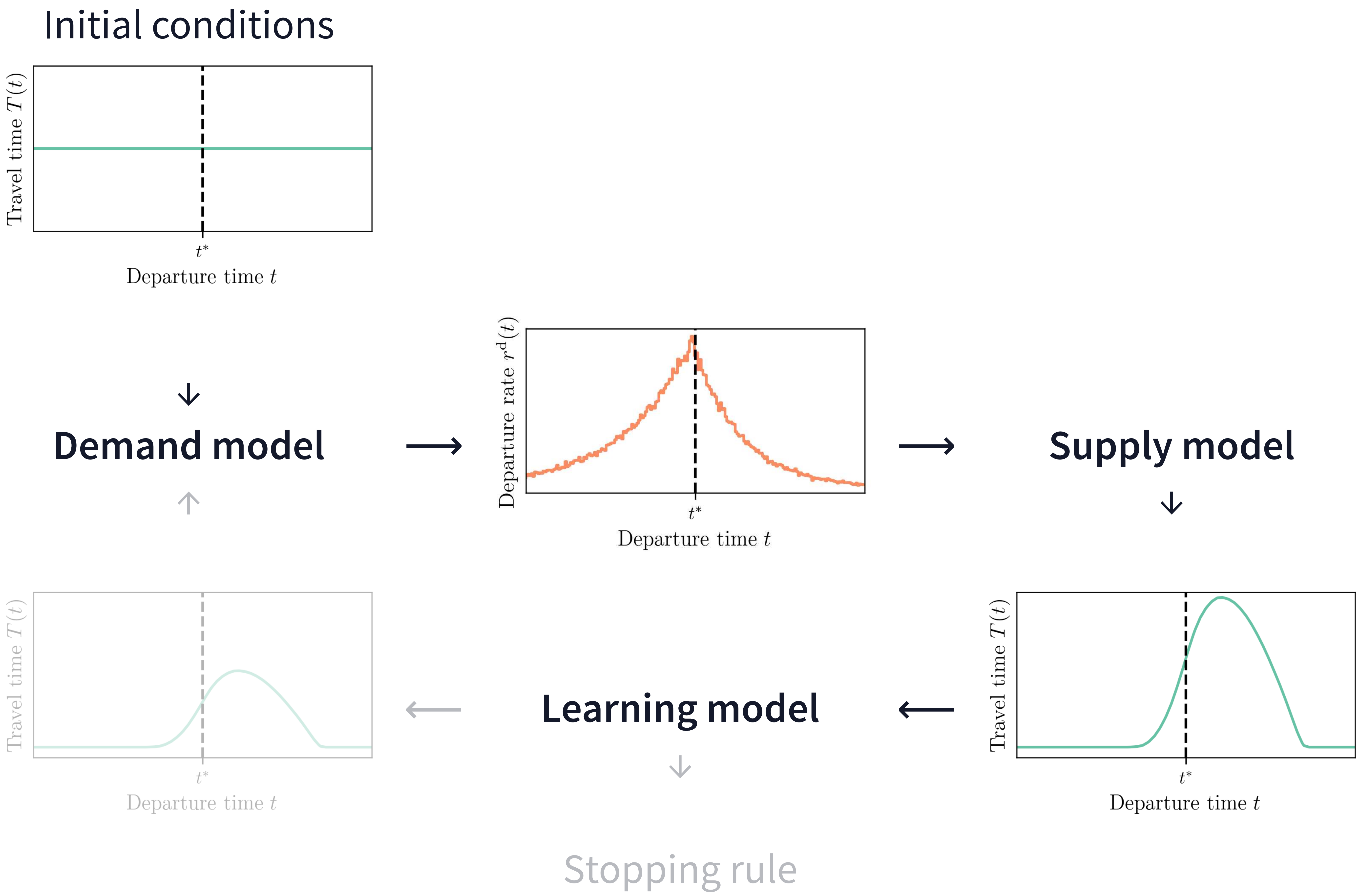

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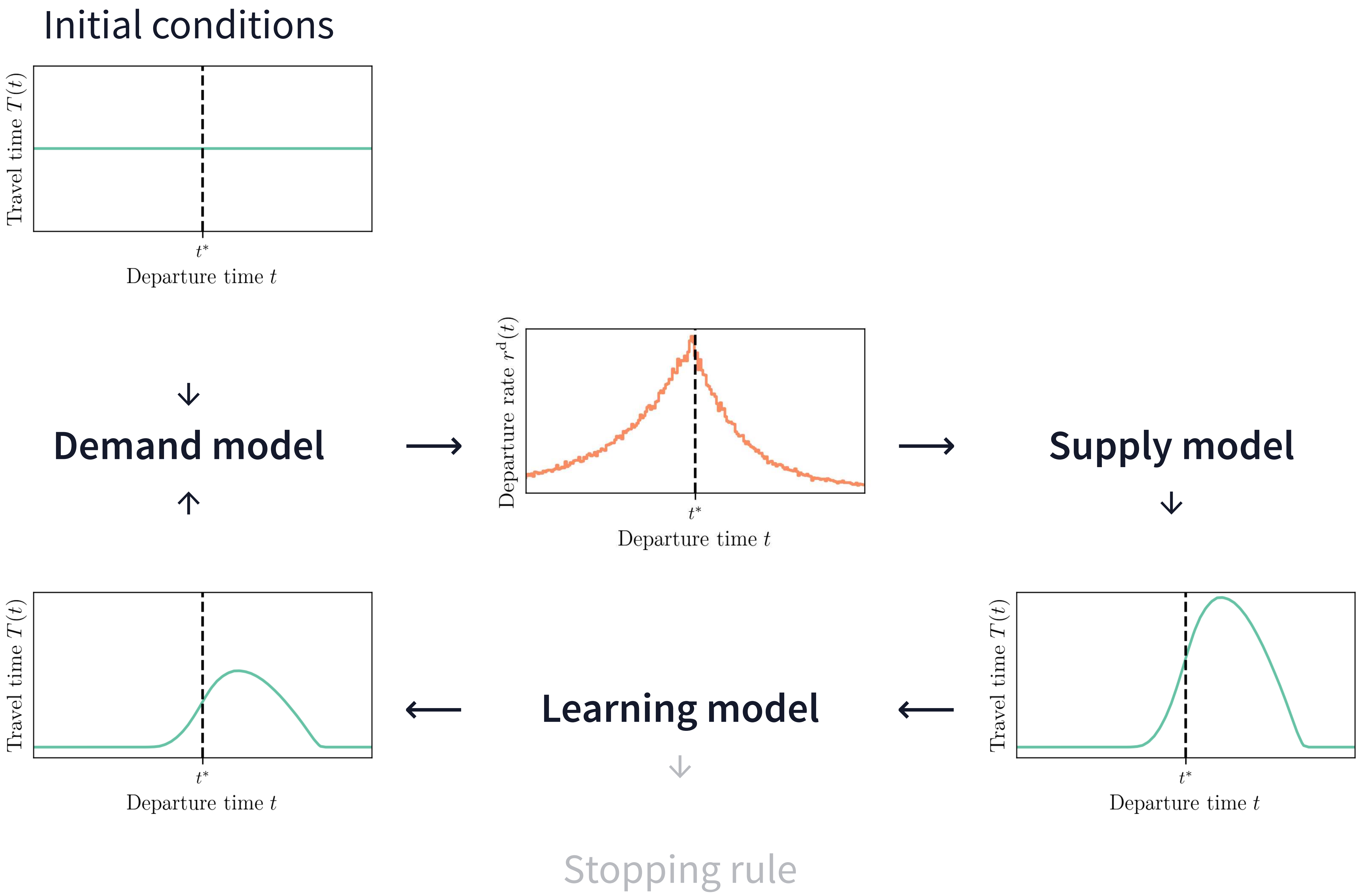

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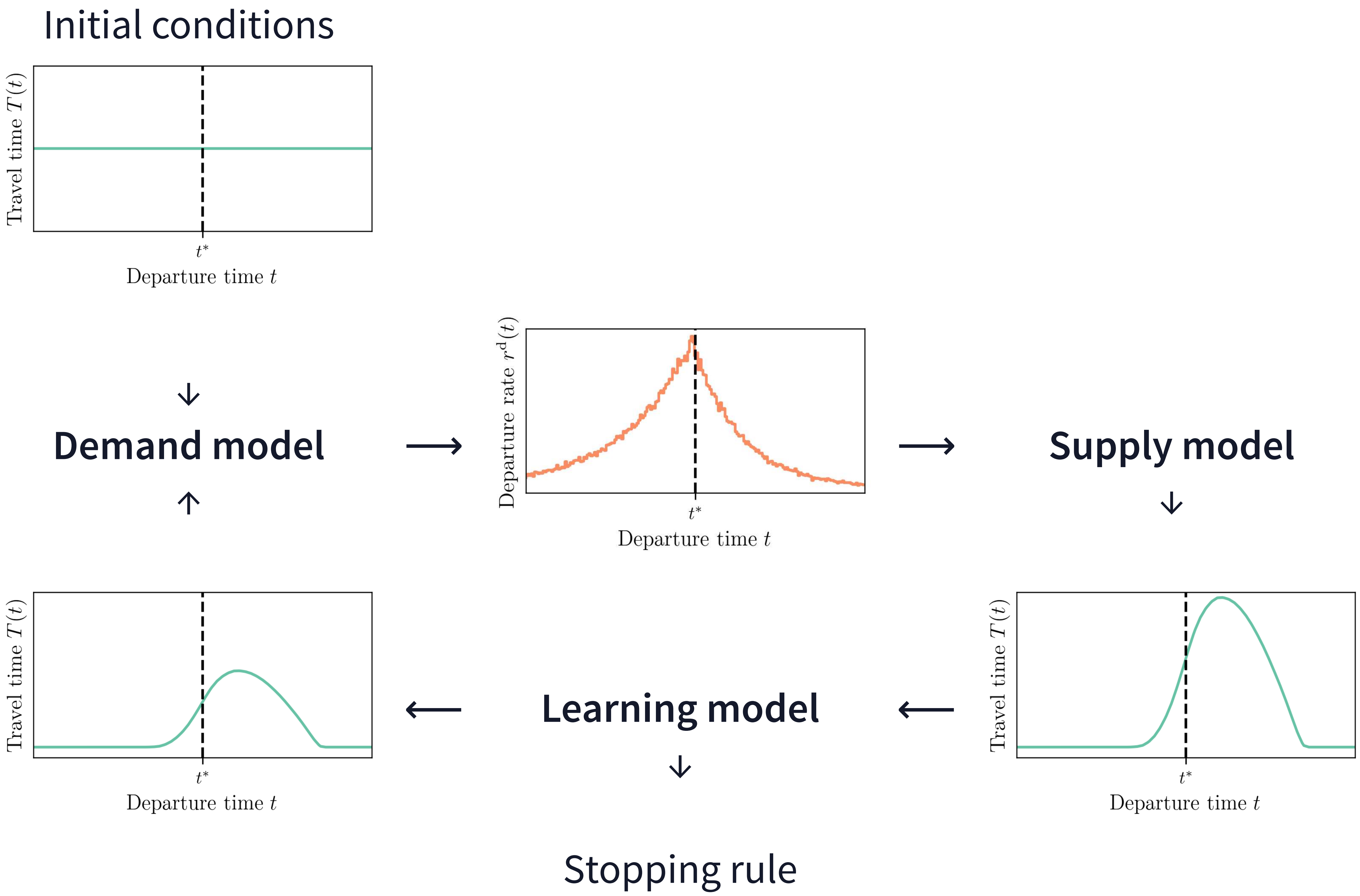

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- Compute **utility function** from the expected travel-time function $\hat{T}$:

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- Compute the departure-time probabilities from the **Continuous Logit formula**:

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- Draw departure times using **inverse transform sampling**

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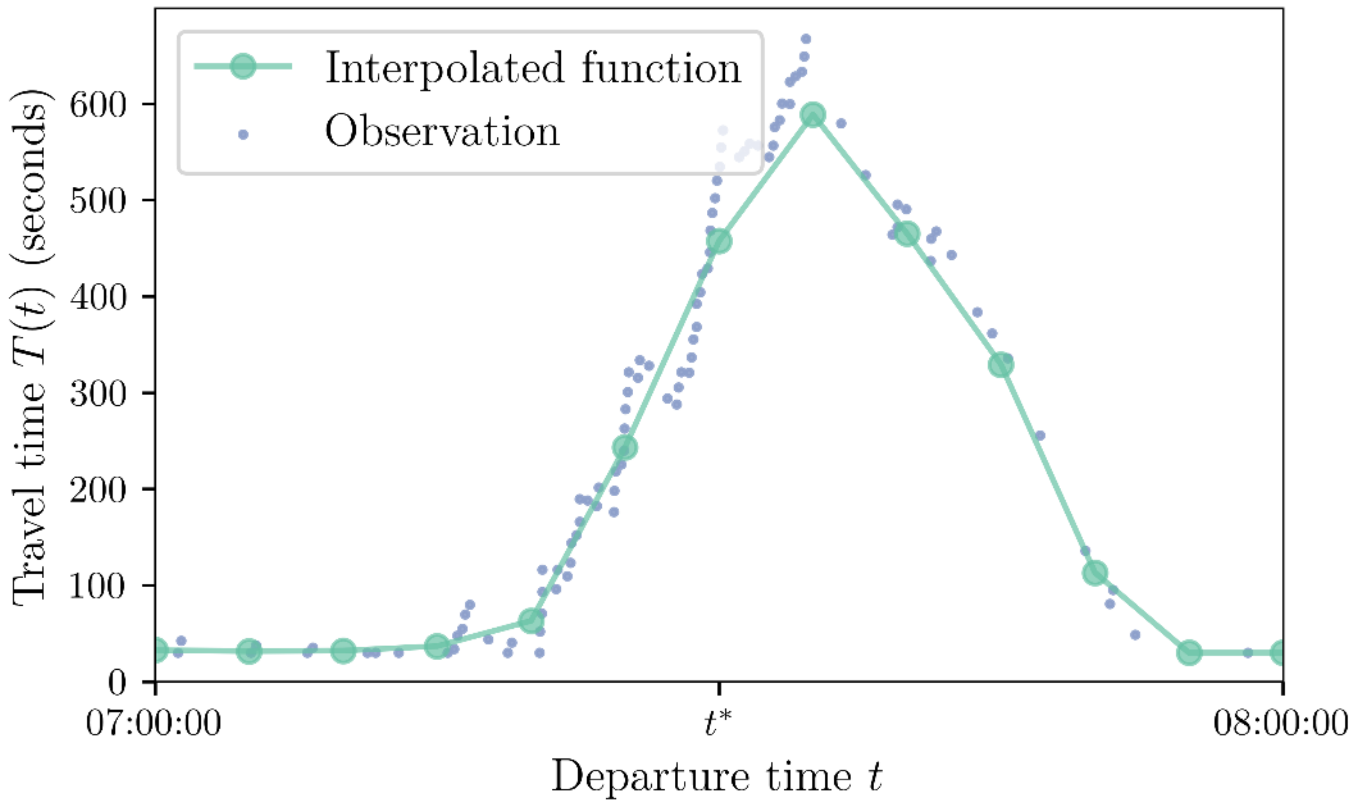
Supply Model

- An **event-based model** is used to simulate the trips of all the agents, from origin to destination
- To simulate a bottleneck of capacity s vehicles per time unit, the road outflow is blocked for $1/s$ time units after each vehicle is crossing
- Example: bottleneck capacity 1800 vehicles / hour $\leftrightarrow$ closing time 2 seconds
- **FIFO:** The cars exit the bottleneck in the same order that they entered it
- **Continuous time:** It can work with very small or very large capacities



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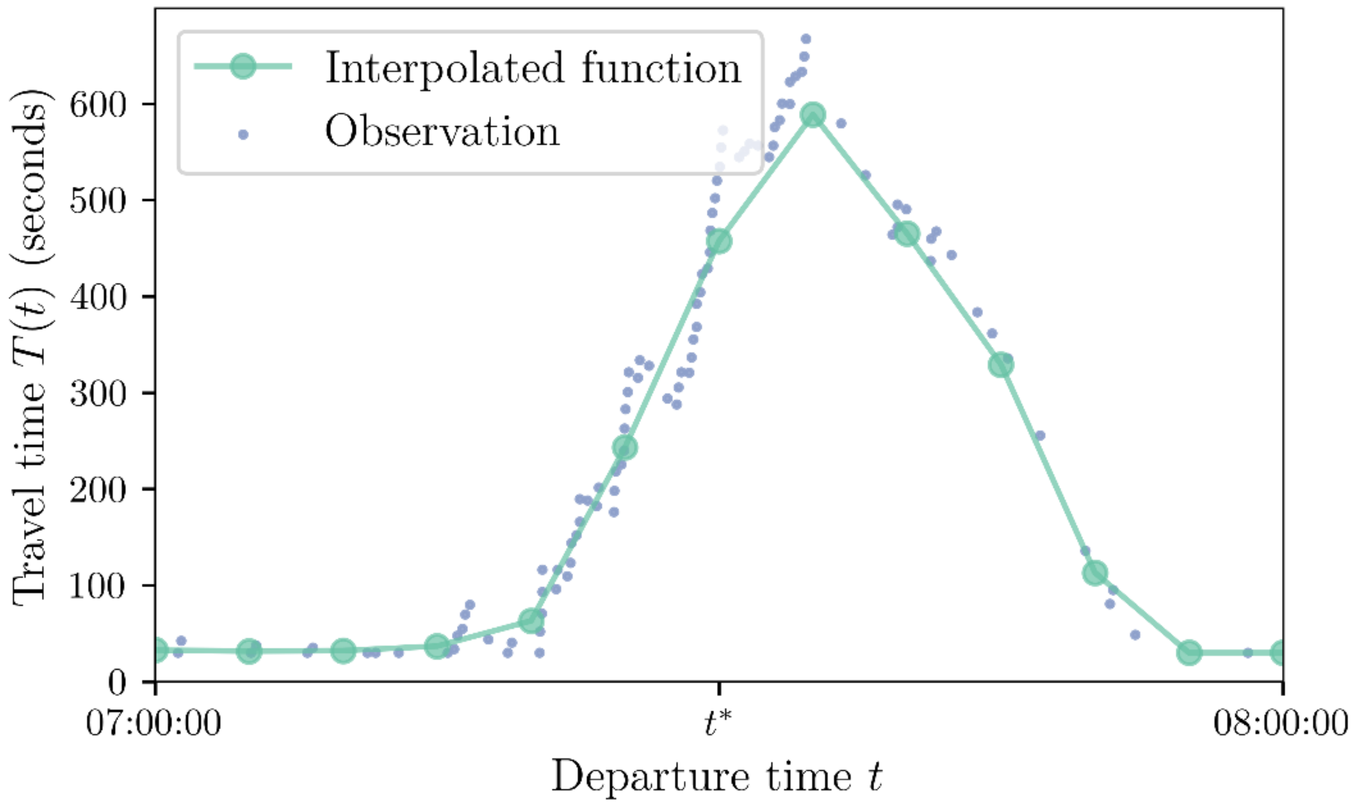

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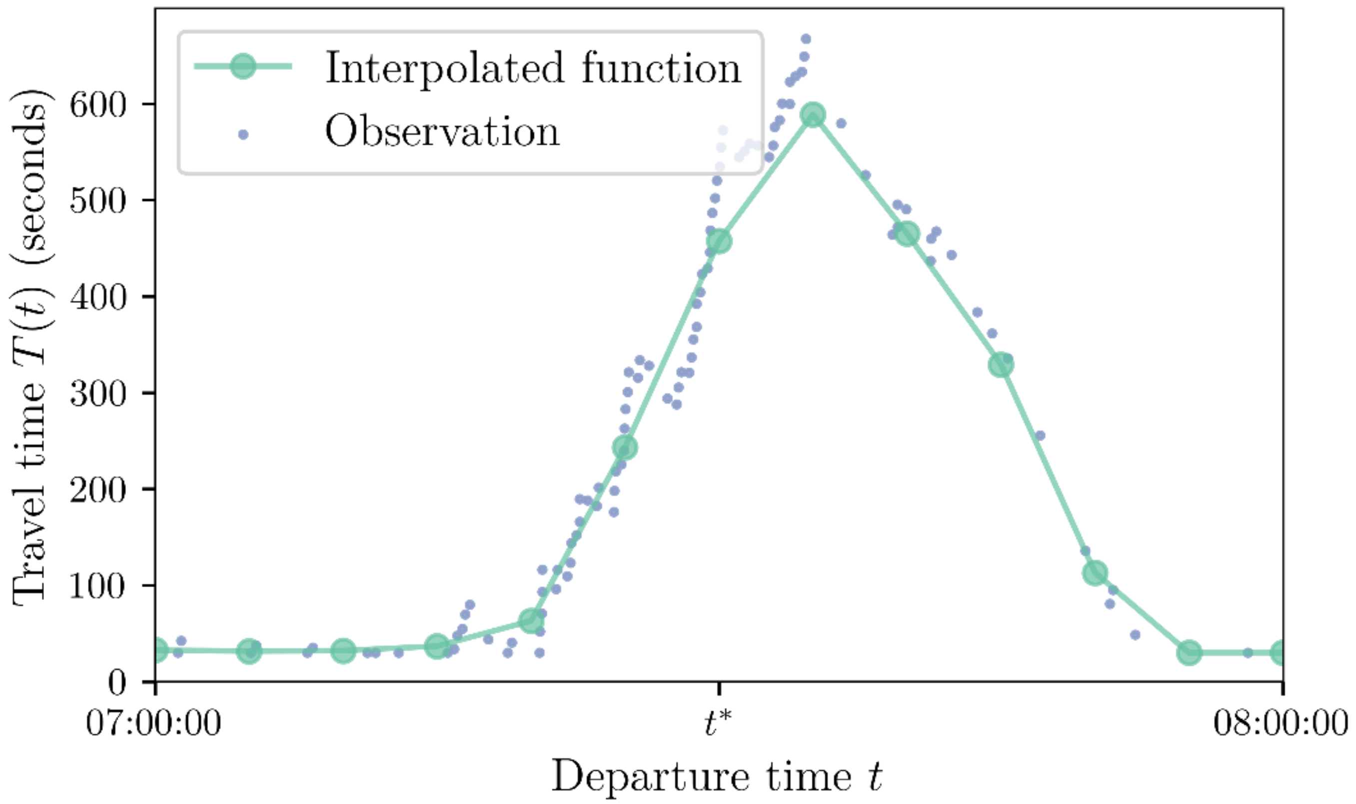

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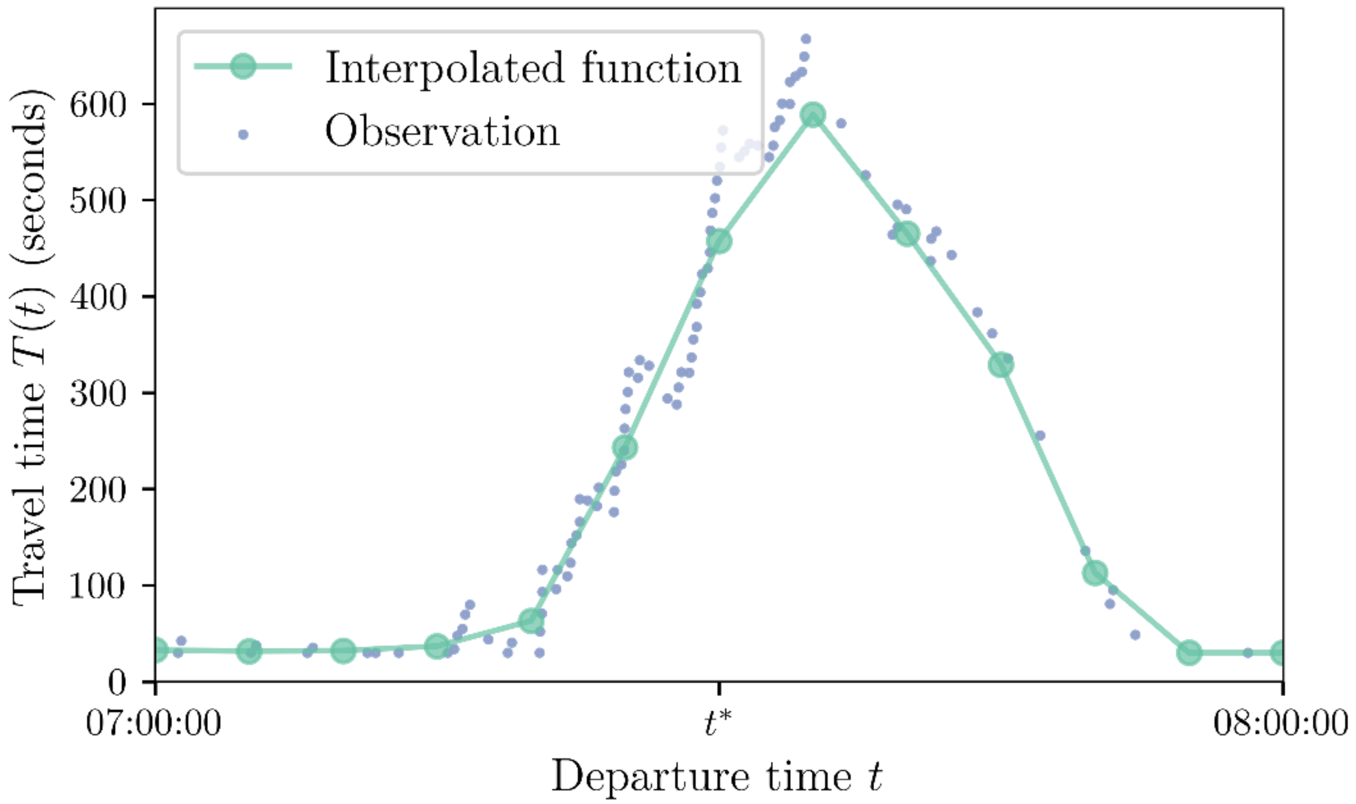

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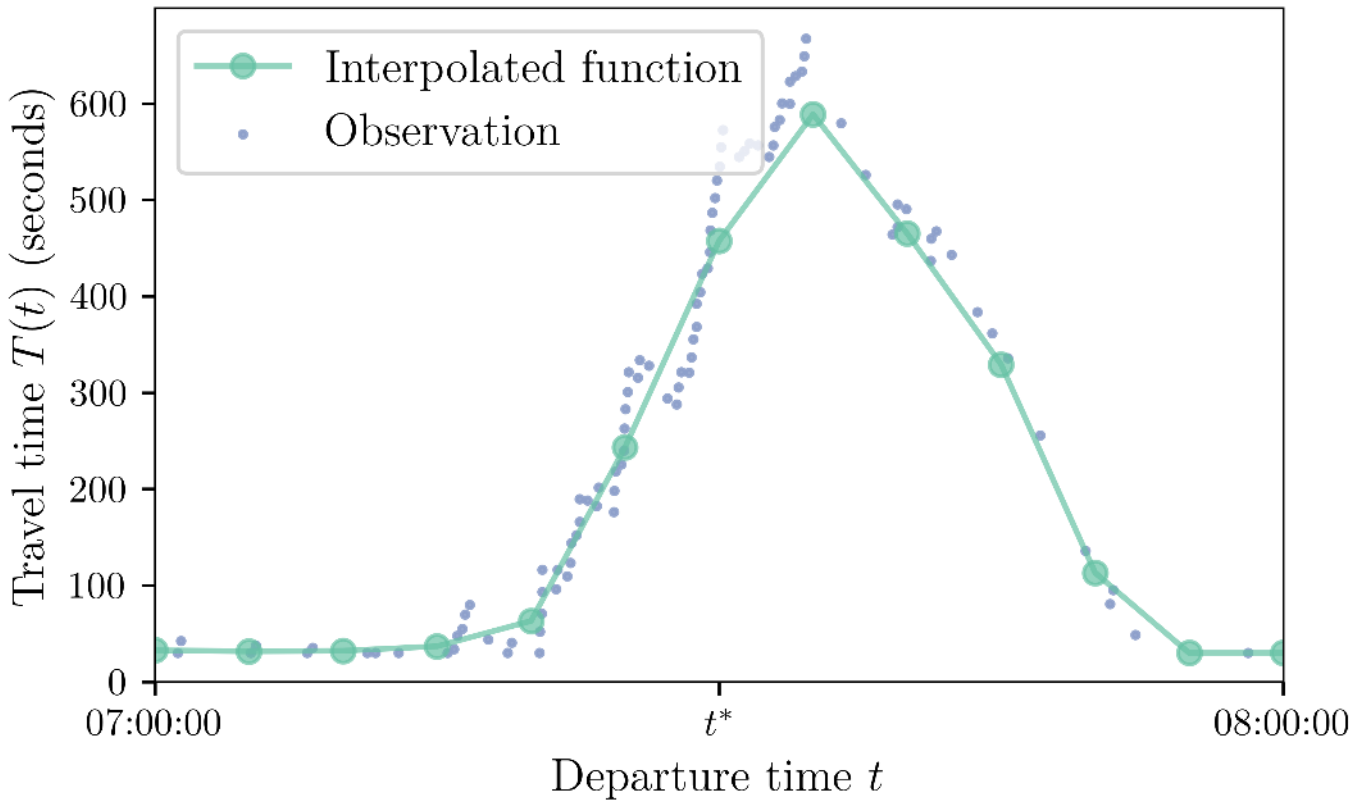
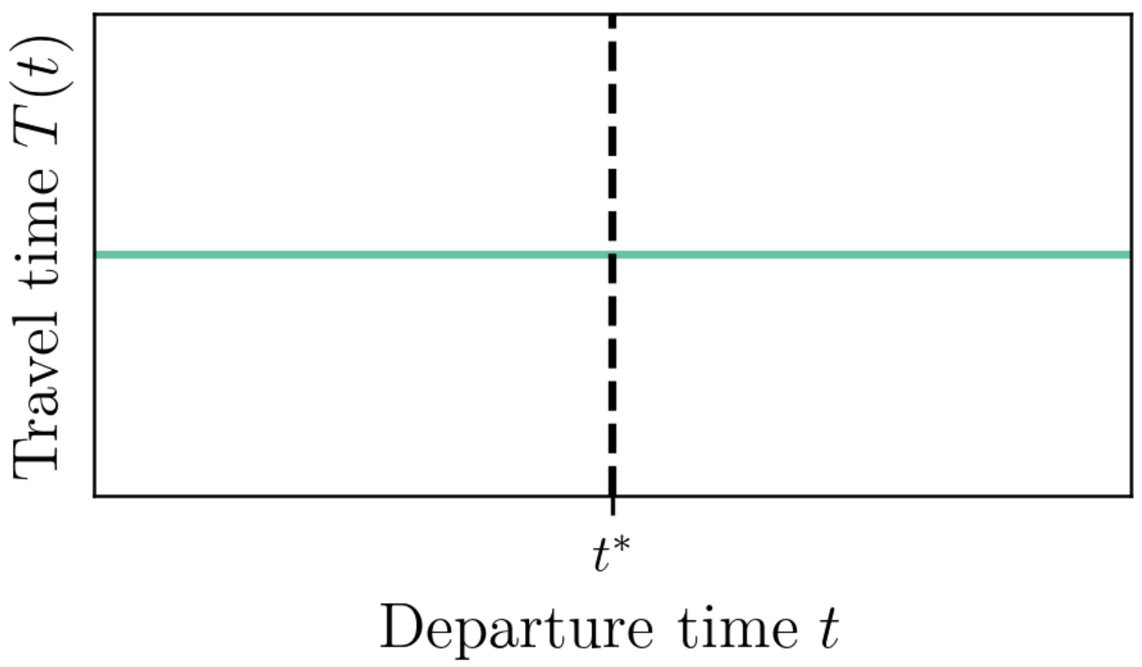


Learning Model

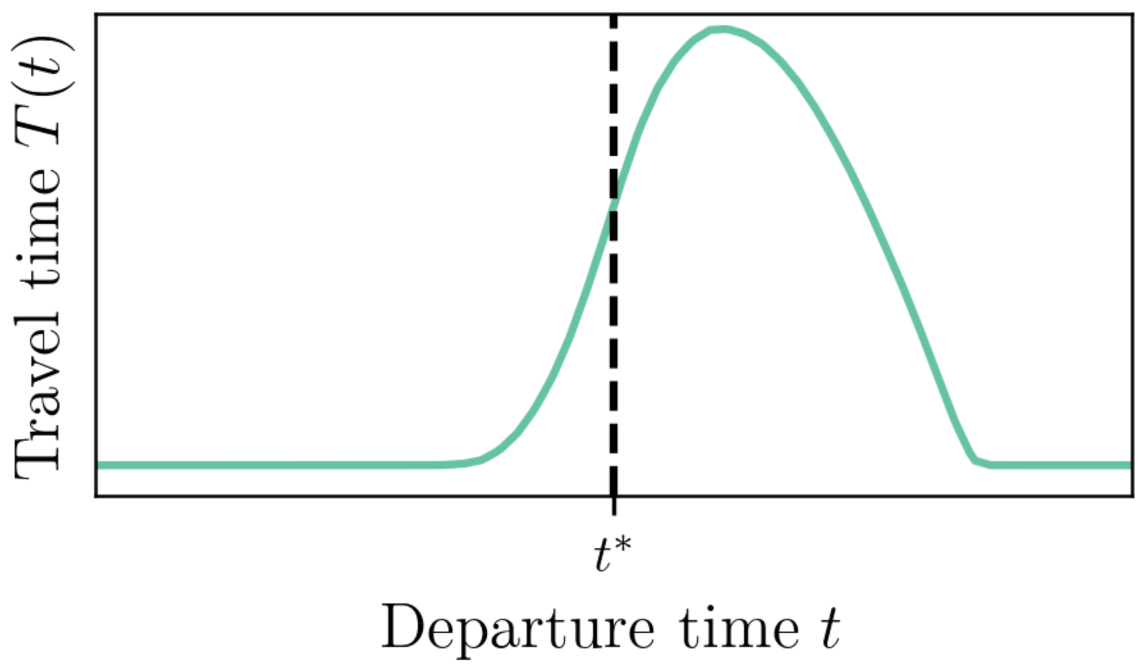
- The travel-time function expected for the next iteration, $\hat{T}_{\kappa+1}$, depends on the simulated travel-time function, T_κ, and the expected travel-time function, $\hat{T}_\kappa$, of the current iteration.
- **Exponential smoothing method:**

$$\hat{T}_{\kappa+1} = \lambda T_\kappa + (1-\lambda)\hat{T}_\kappa$$

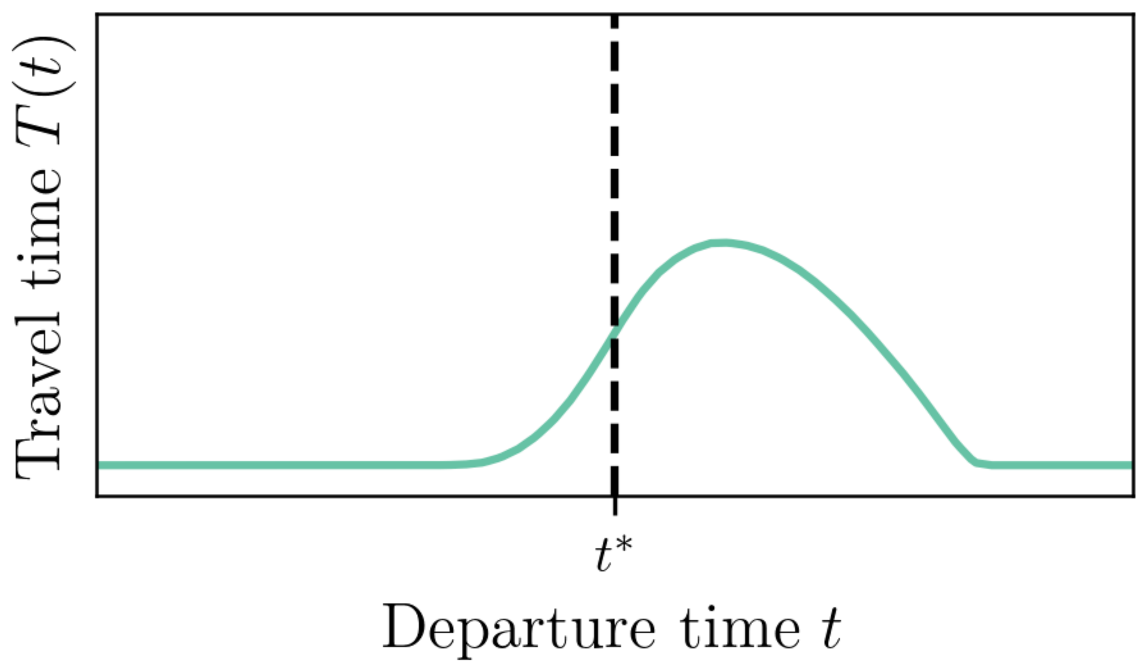
  with $\lambda \in [0, 1]$, the smoothing factor.
- An equilibrium is reached when $\hat{T}_\kappa = T_\kappa$



$\hat{T}_\kappa$



T_κ



$\hat{T}_{\kappa+1}$

Learning Model

- The travel-time function expected for the next iteration, $\hat{T}_{\kappa+1}$, depends on the simulated travel-time function, T_κ, and the expected travel-time function, $\hat{T}_\kappa$, of the current iteration.
- **Exponential smoothing method:**

$$\hat{T}_{\kappa+1} = \lambda T_\kappa + (1-\lambda)\hat{T}_\kappa$$

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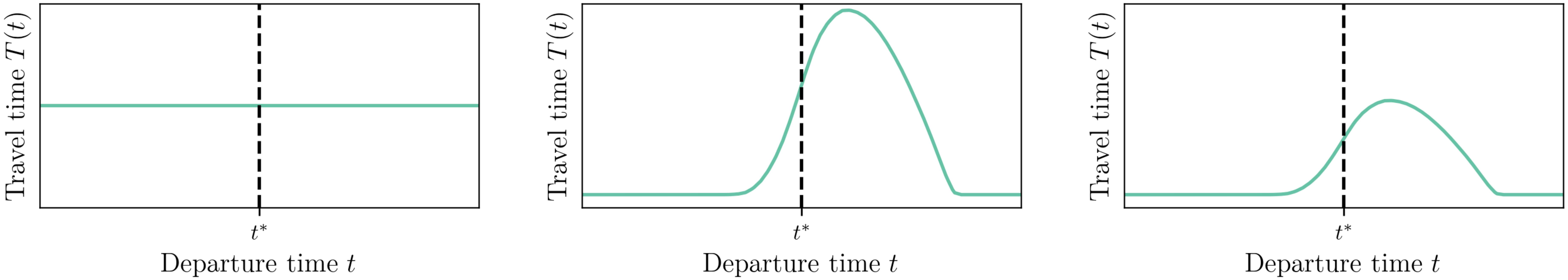

$\hat{T}_\kappa$ T_κ $\hat{T}_{\kappa+1}$

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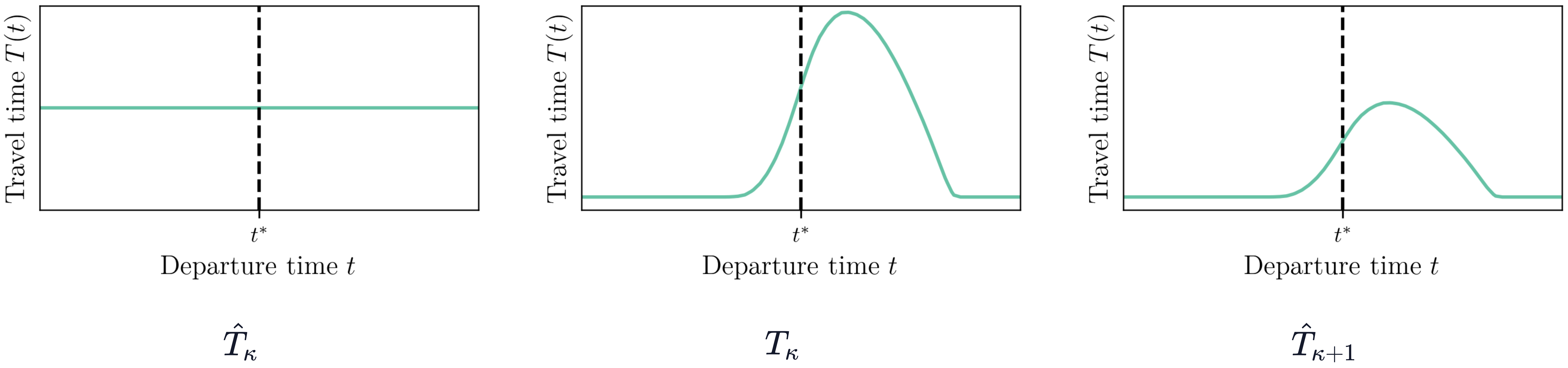
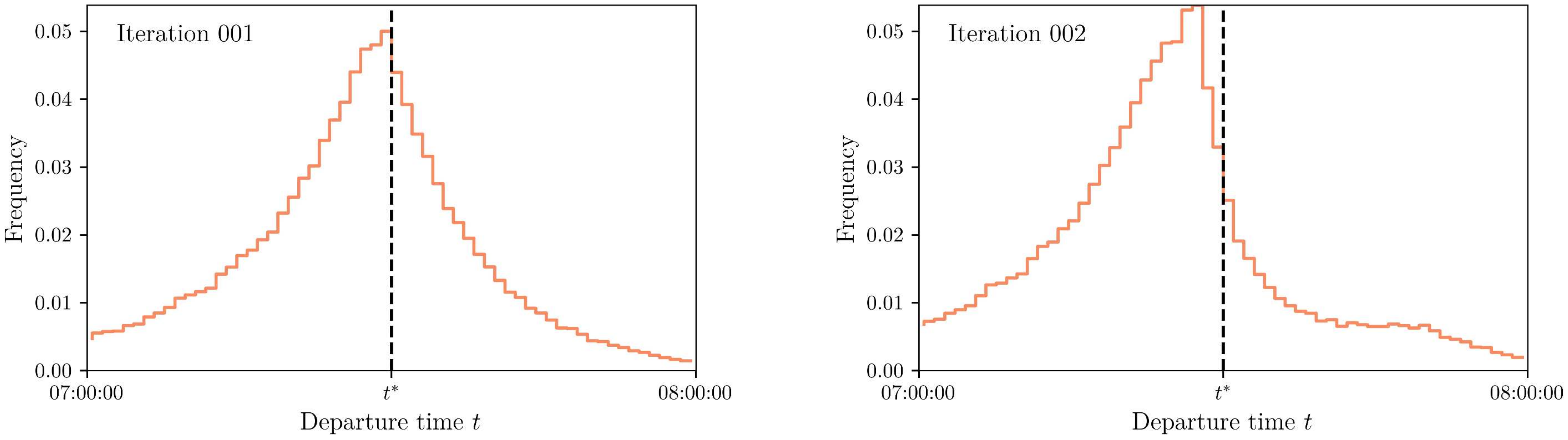


Results

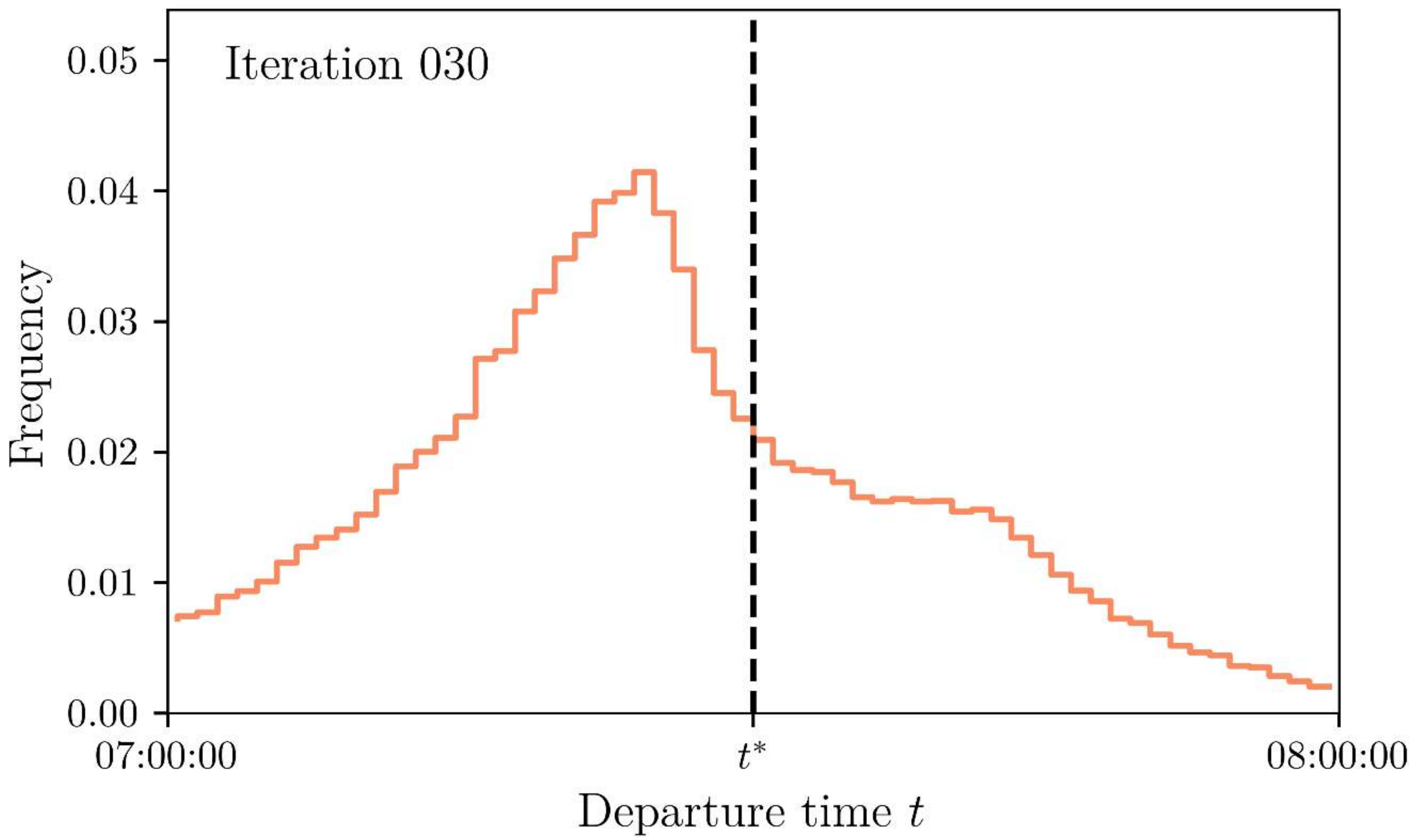
Simulations

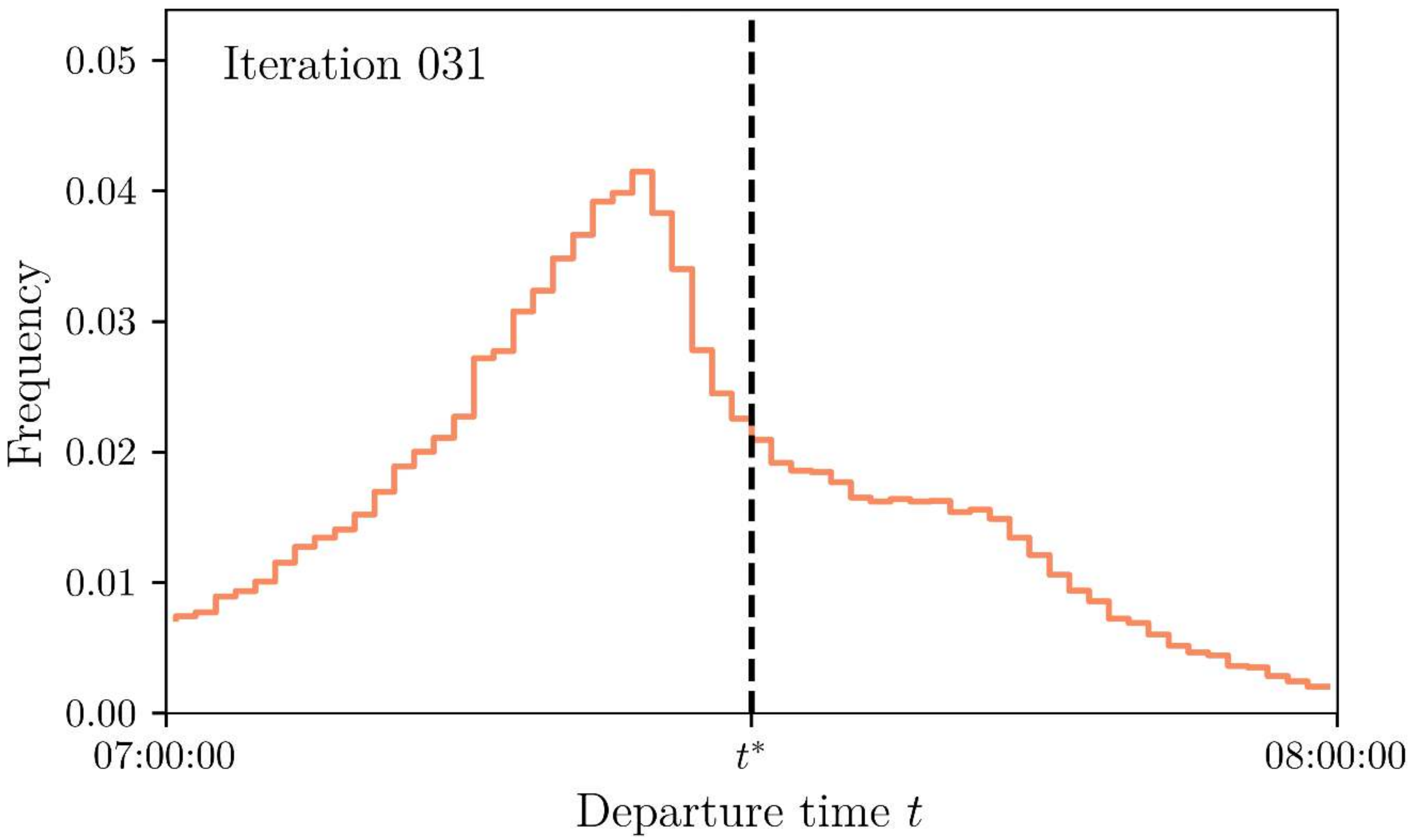
- $N = 100,000$ agents
- $\alpha = 10\$/h, \beta = \gamma = 5\$/h$
- t^* = 7:30 a.m.
- Free-flow travel time t^f = 30 seconds
- Bottleneck capacity $s = 150,000$ cars / h
- Smoothing factor $\lambda = 0.5$

Departure rate: Iteration 1 & 2

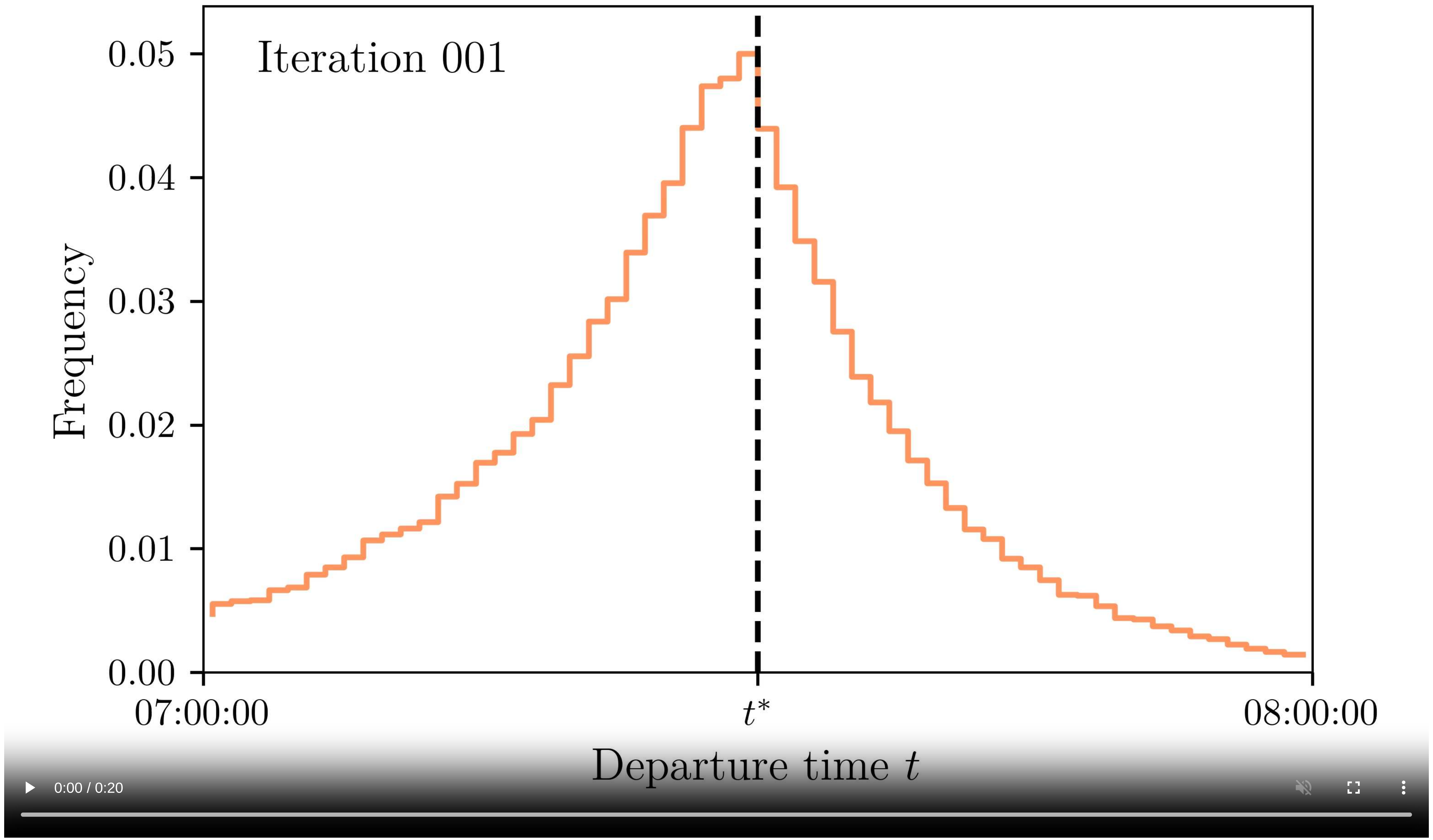


Departure rate: Iteration 30 & 31



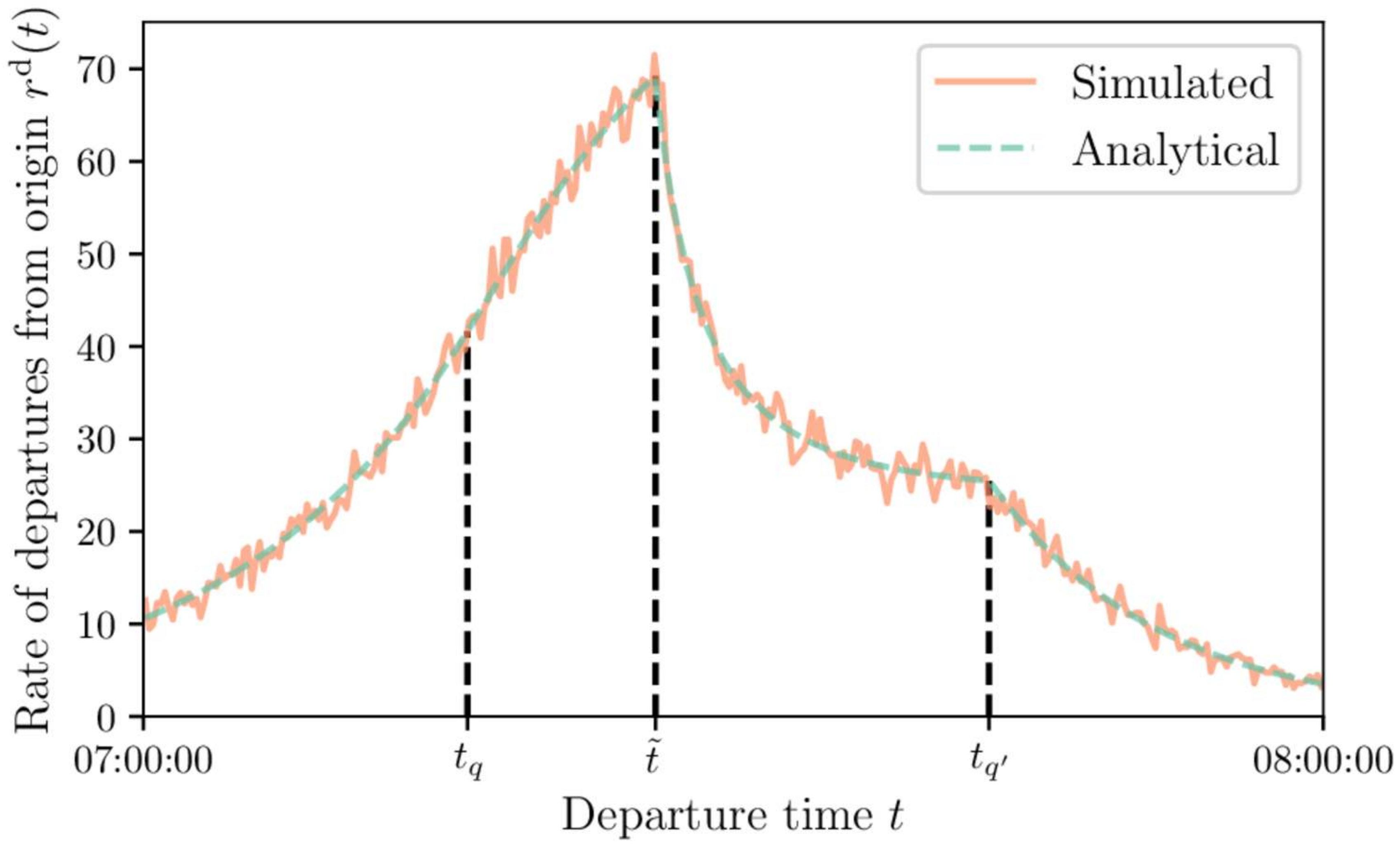


Convergence Video

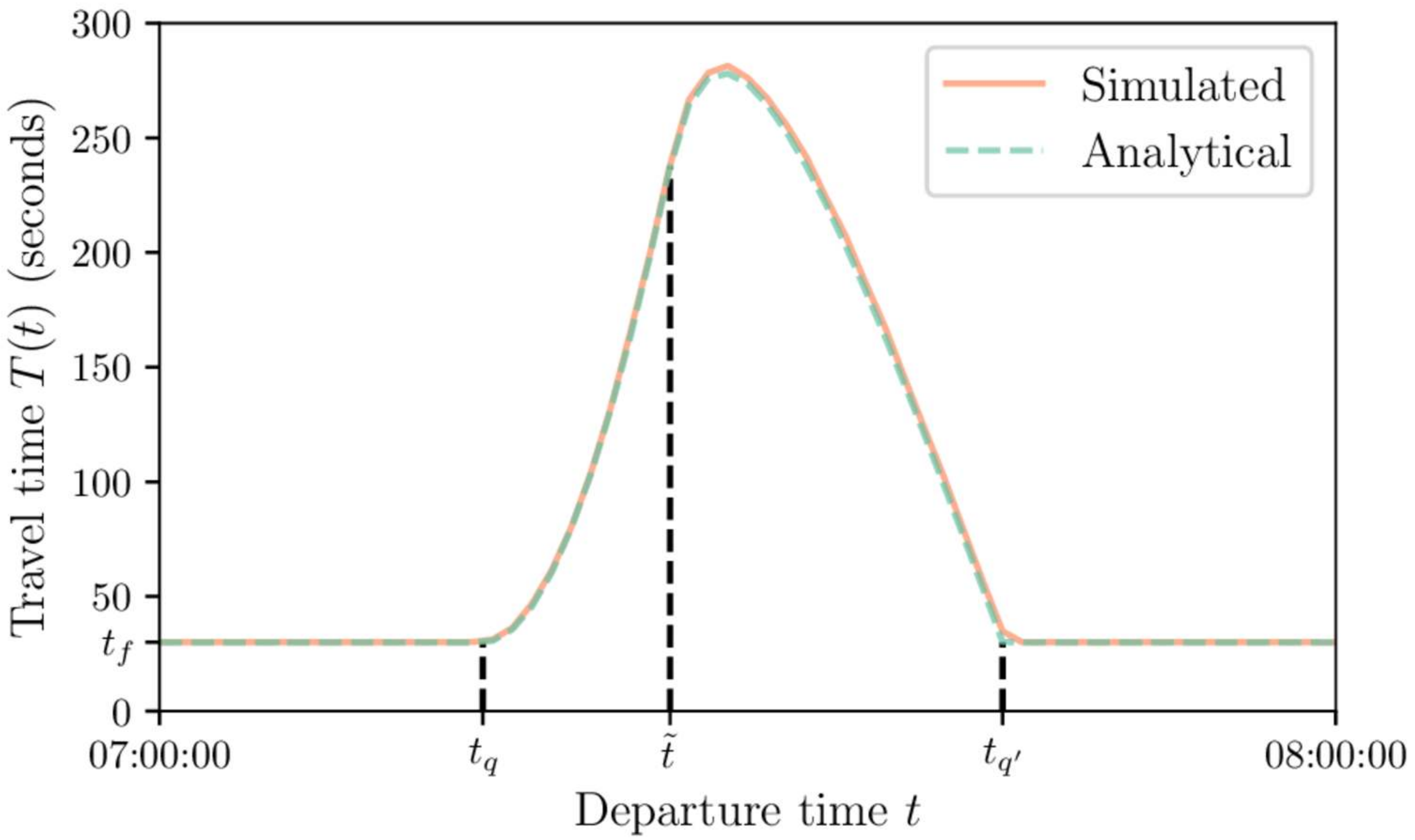


Comparison to Analytical Results

Departure-time distribution



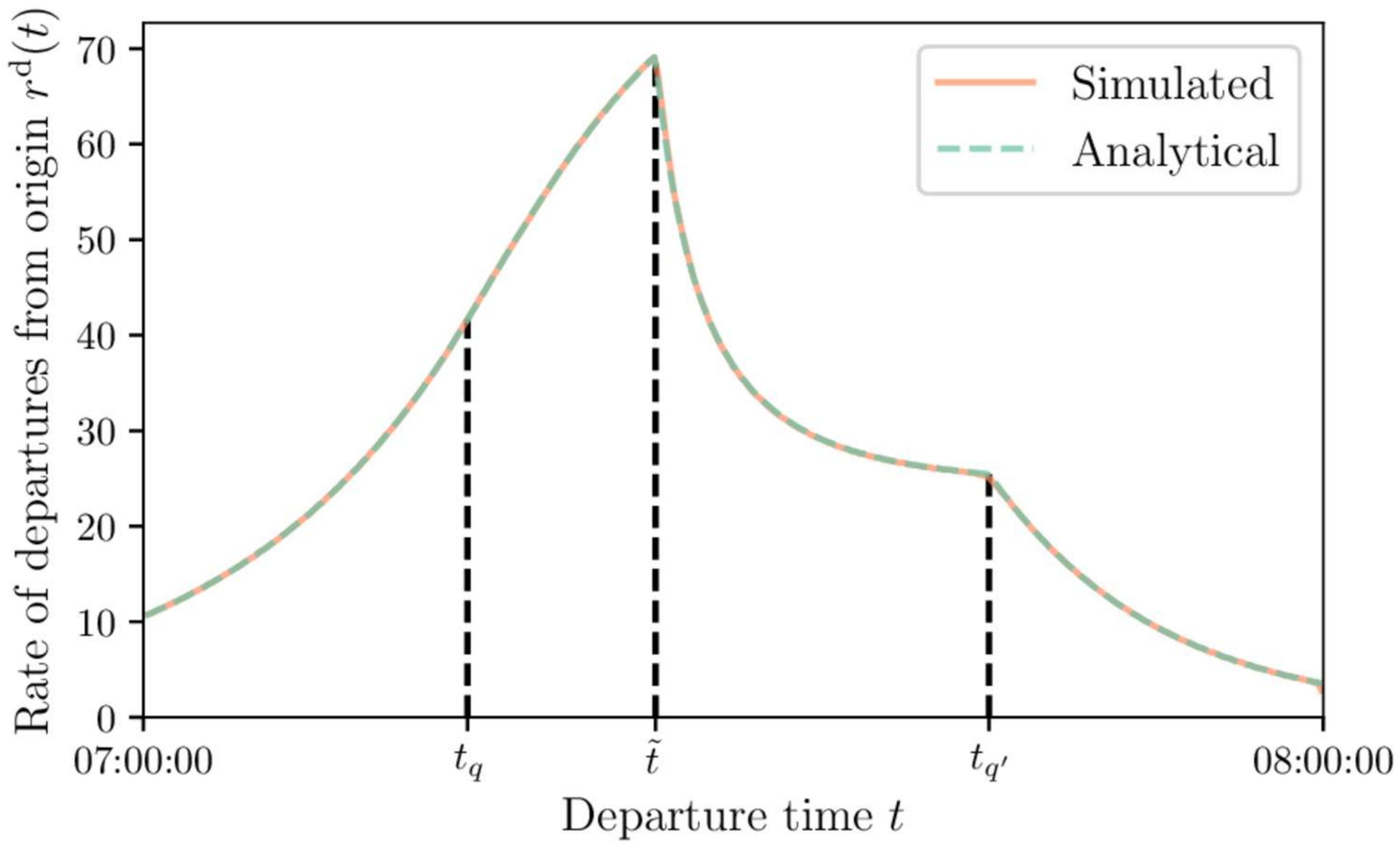
Travel-time function



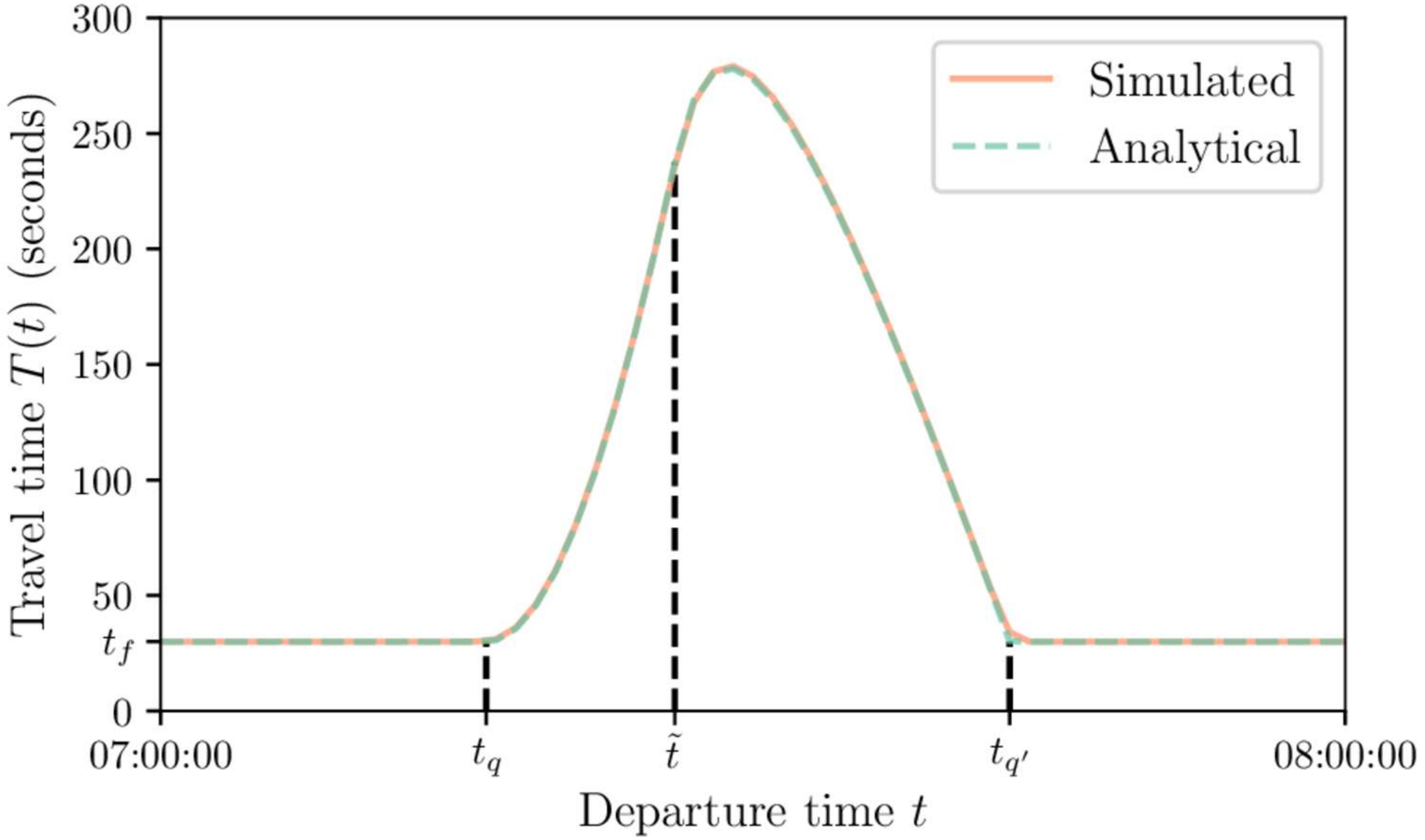
Systematic Sampling

Systematic sampling: using evenly spaced values in $[0, 1]$ instead of random draws

Departure-time distribution



Travel-time function



Large-Scale Simulations

METROPOLIS2: Large-Scale Supply Side

- **Road network:** arbitrary graph of nodes (intersections) and edges (road links)
- **Congestion:** link-level bottleneck and speed-density function, queue propagation (spillback)
- **Vehicle types:** headway, speed limits, road restrictions

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- **Mode choice:** arbitrary number of modes (congested and uncongested)
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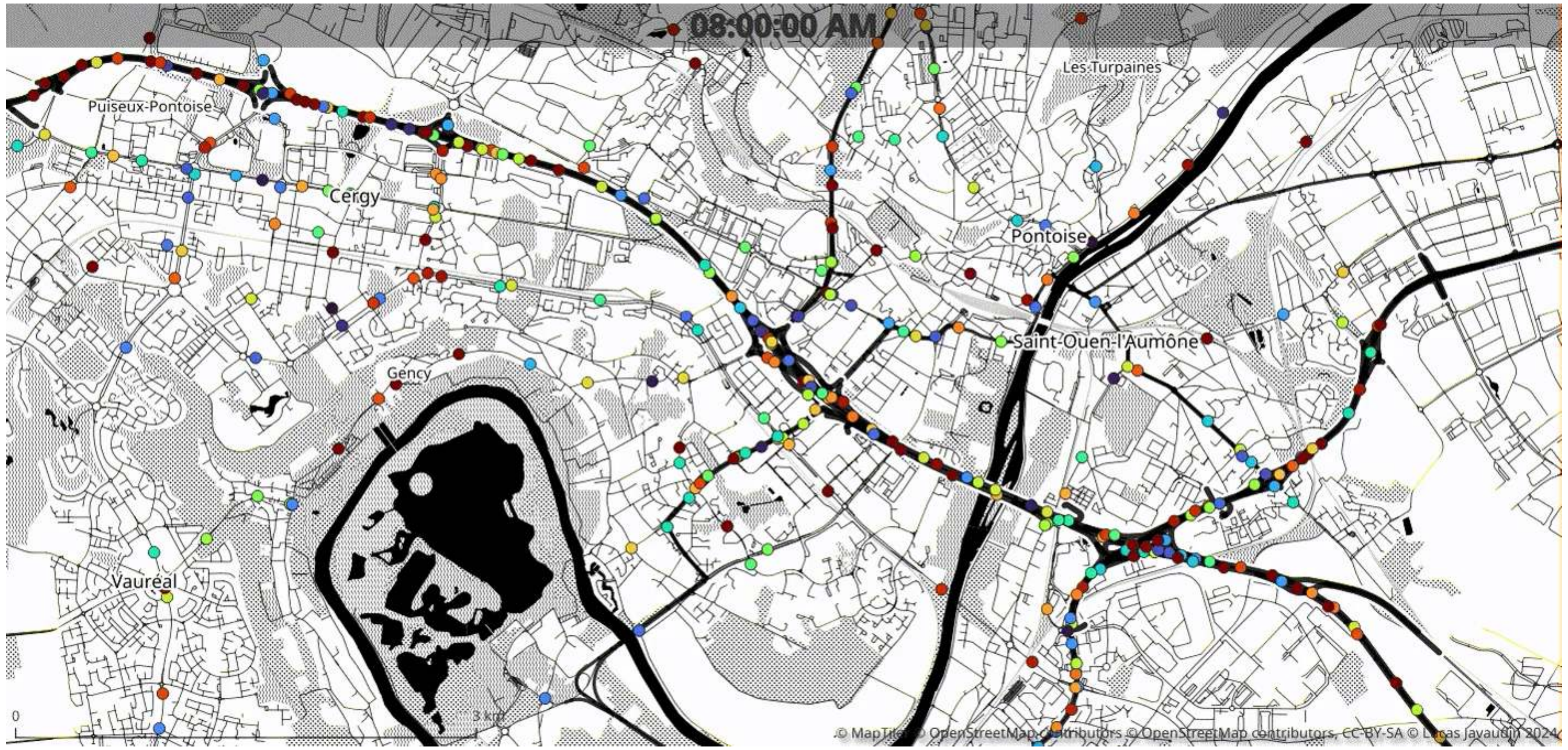
Example Application: Paris' Urban Area

	METROPOLIS1	METROPOLIS2
Running time	18h 29m	1h 49m
RMSE departure time	2m 5s	5s
Average utility	-5.58 €	-5.32 €
Average travel time	16m 23s	15m 33s



Conclusion

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- We proposed a **simulation methodology** to solve some analytical models
- The methodology can be used to solve models which are **too complex** to be derived analytically (e.g., toy network, heterogeneous t^*)
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Thank you

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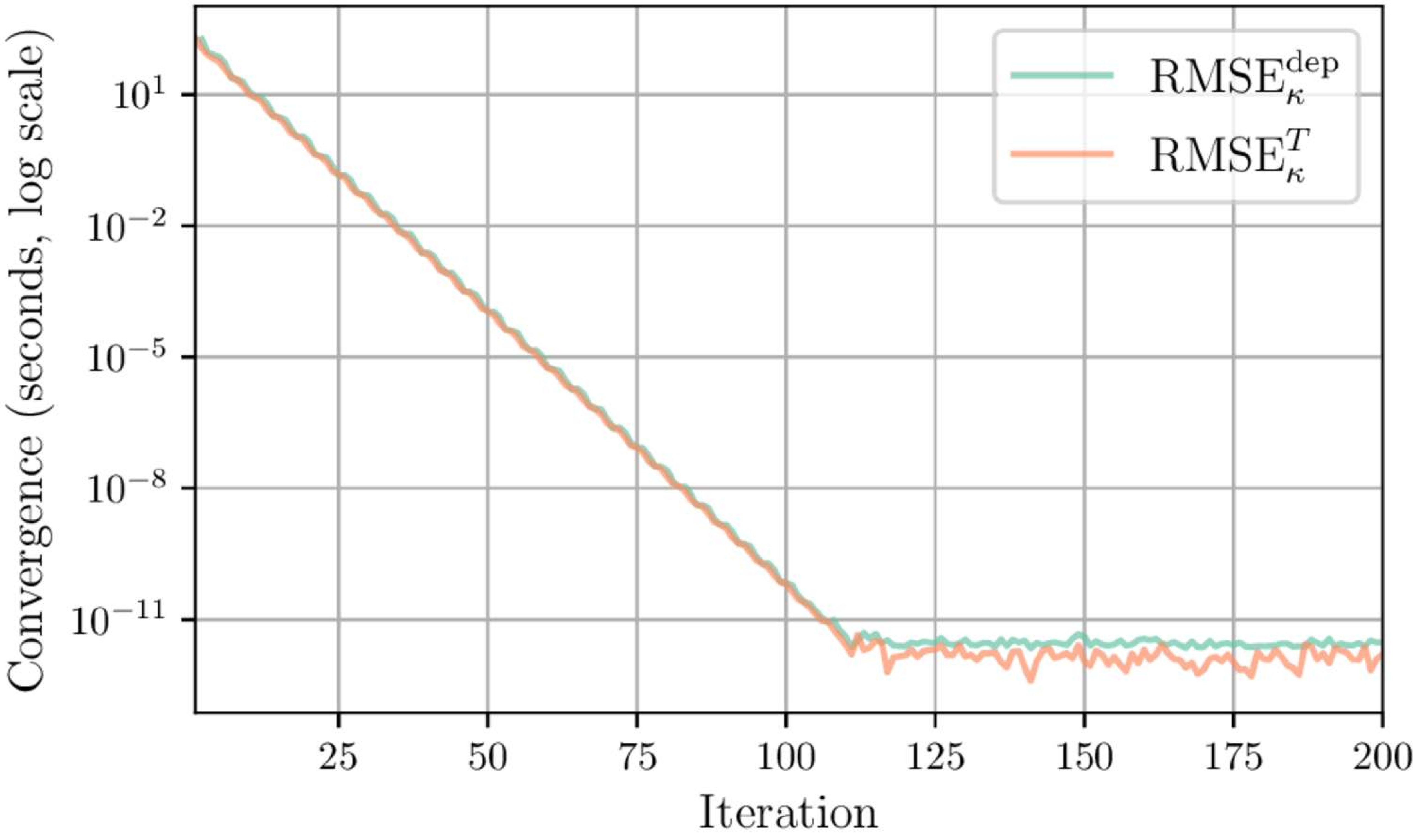
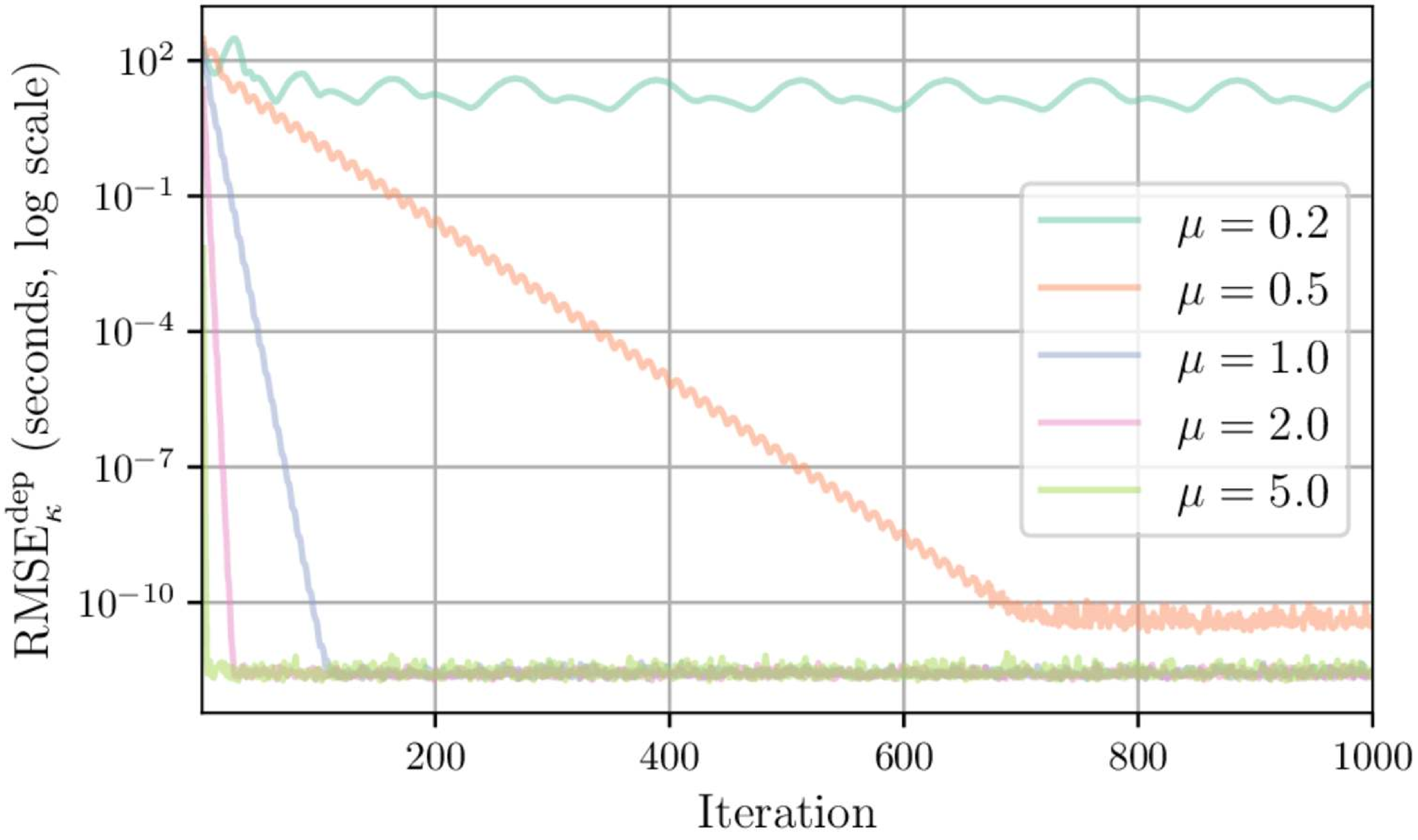
Appendix

Convergence



Impact of utility scale on convergence



Heterogeneous desired arrival times